Geometry and Reality

perspective in cubism und quantum mechanics

Talk Gerd Breitenbach, Dornach, Februar 2015

What is the „noumenon“ behind the phenomena?

Method of projective geometry:
Investigate not only the object but preeminently the laws of visual perception

Geometry and Reality

perspective in cubism und quantum mechanics

Perspective in the Renaissance

Unambiguity of reality

Counter example: Theorem of Desargues

Classical physics:

State of a system in phase space

Quantum mechanics:

The measurement process as a projection: The Wignerfunction

Example: light/atom as waves/particles

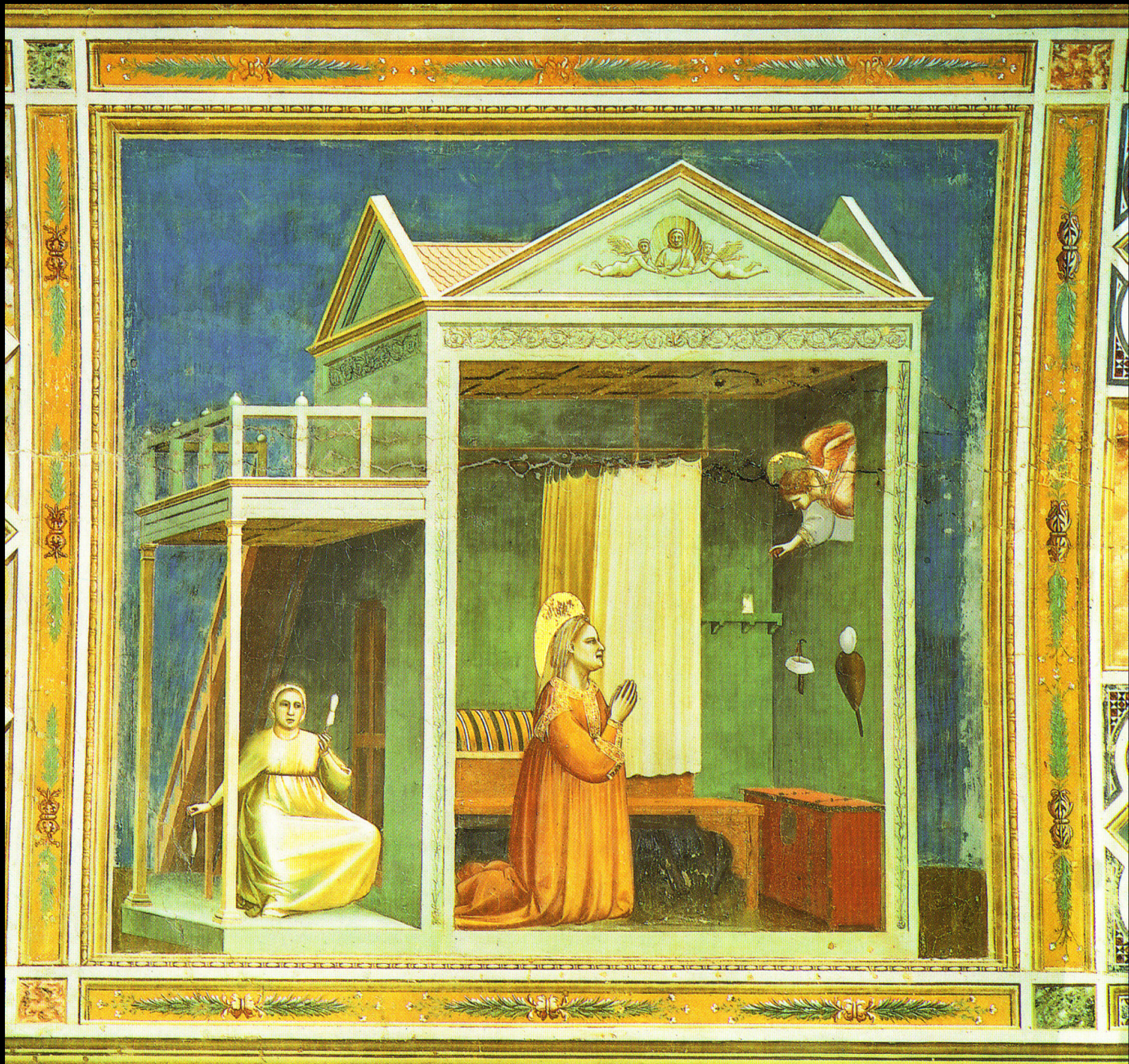
Duality in projective geometry and quantum mechanics

The conceptual change of perspective

Perspective in cubism

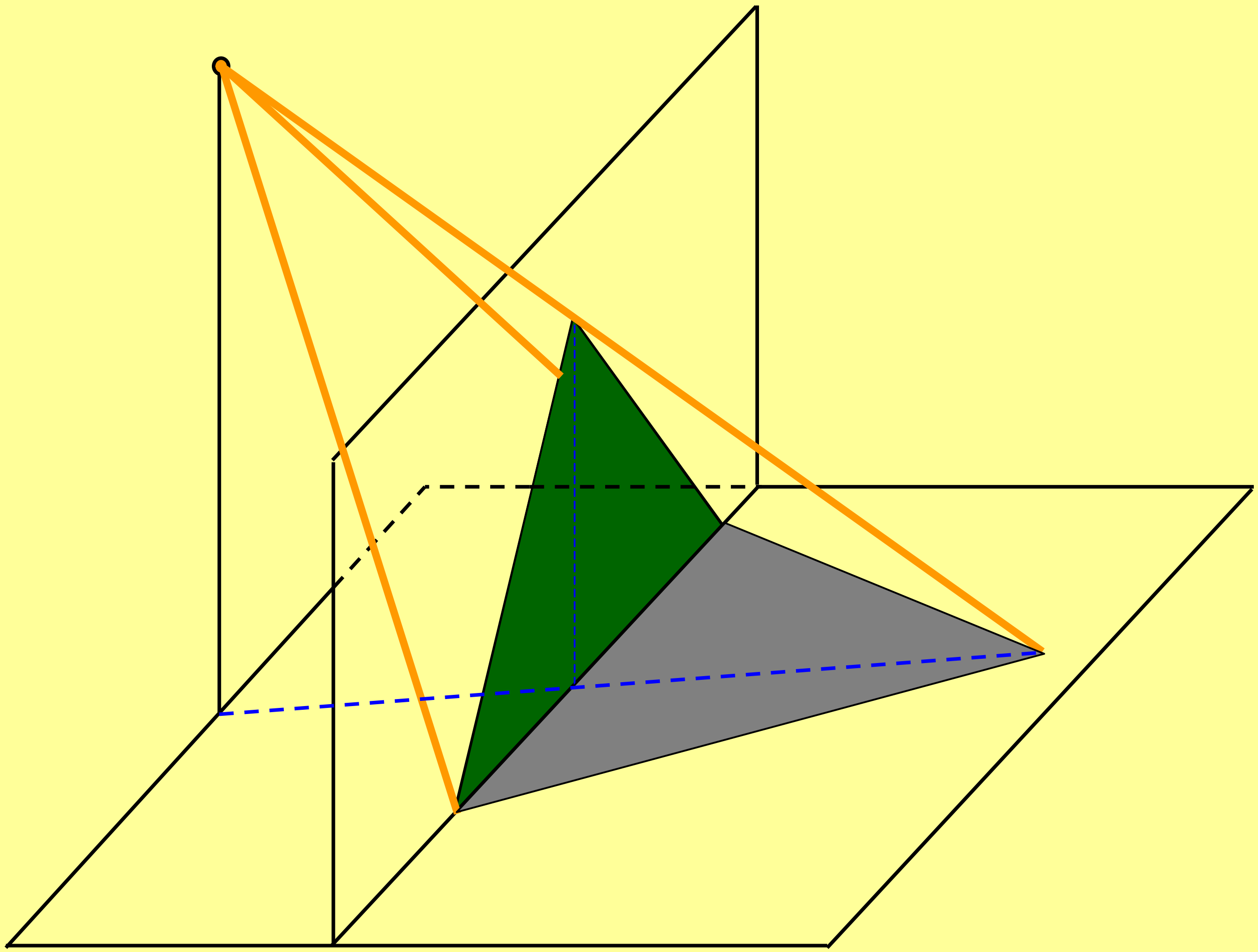
The ambiguity of reality

Perspective in the Renaissance

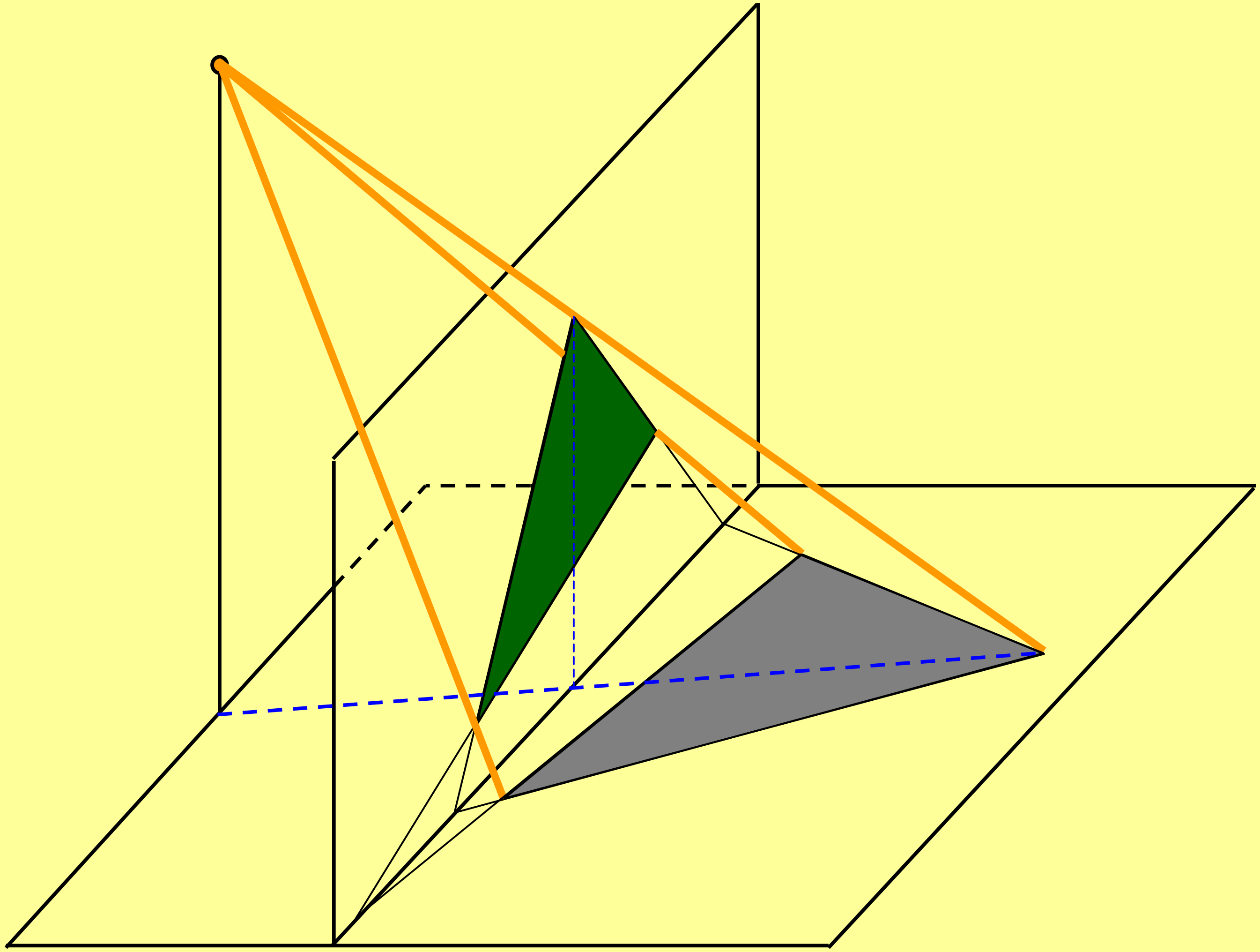


Giotto (1267-1337)
The Annunciation of St. Anna
Arena chapel, Padua, 1303

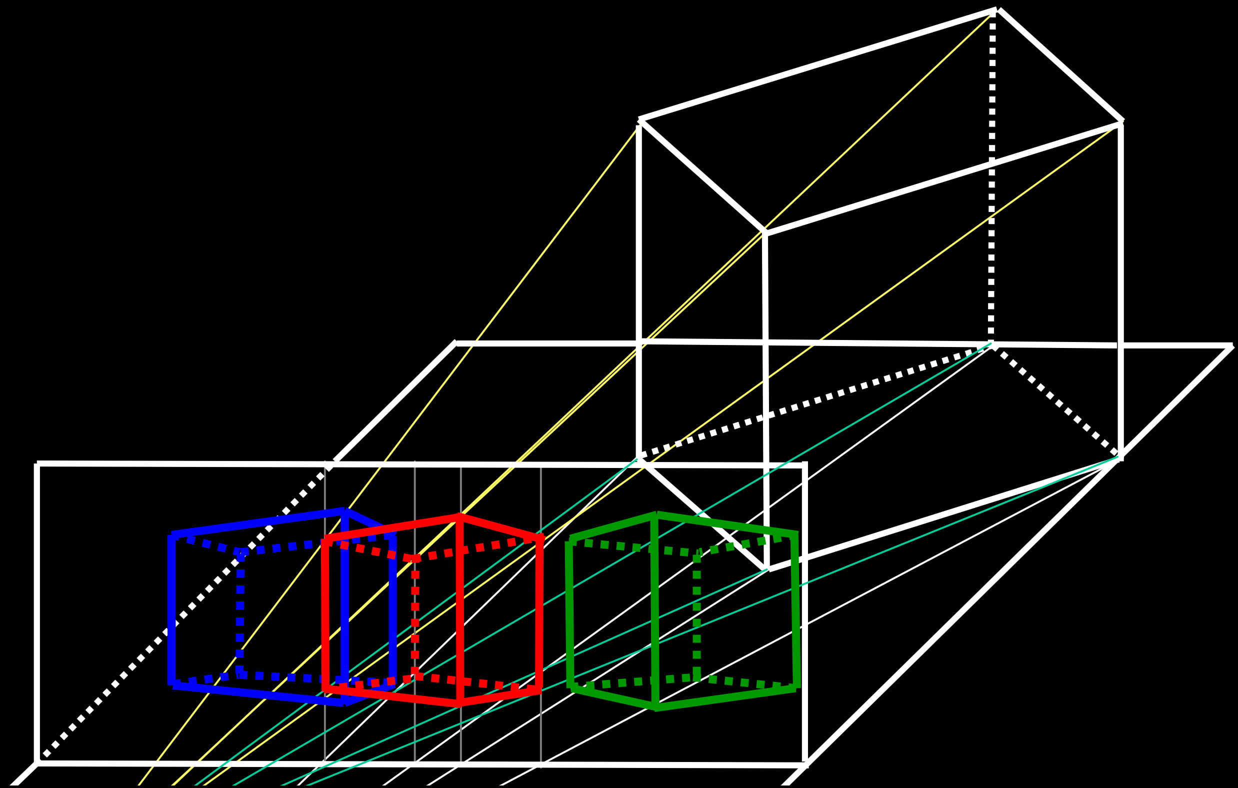
Shadow casting



Shadow casting



Canvas projection

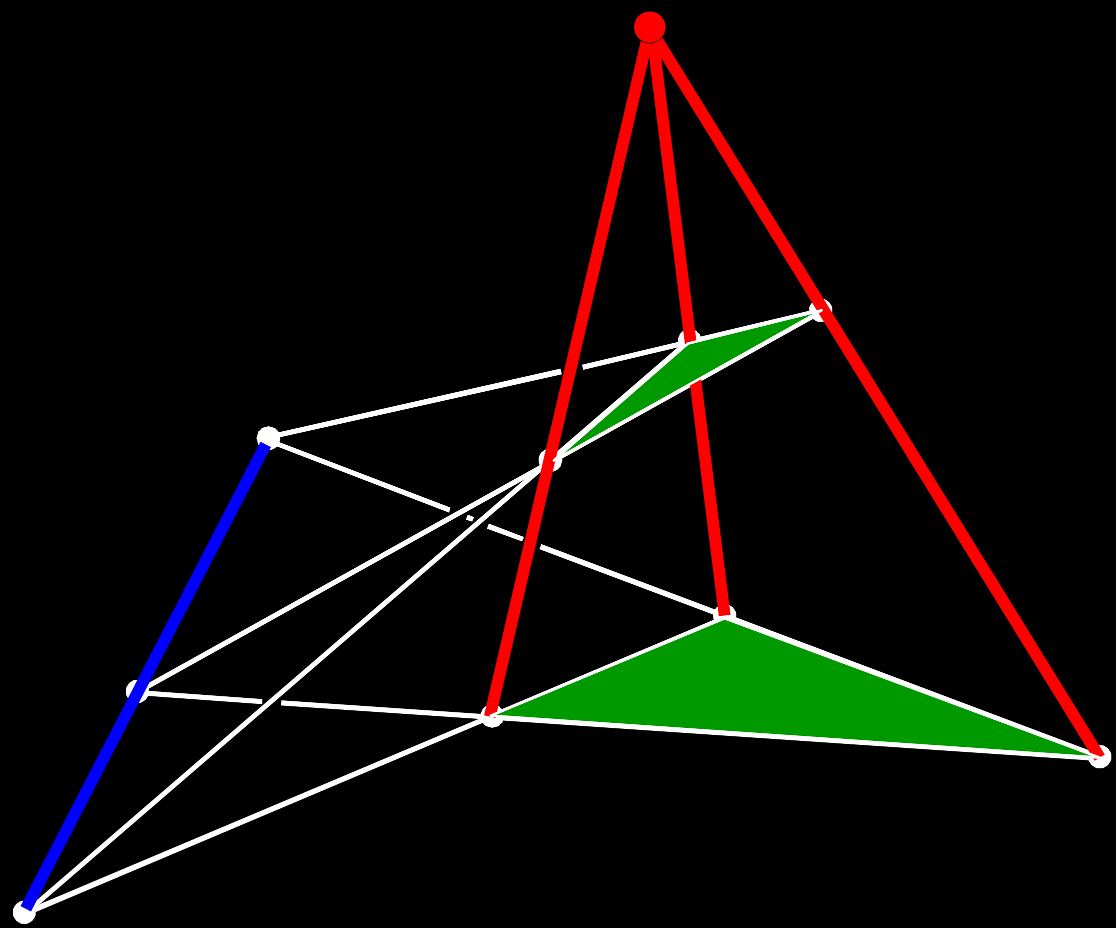


Rekonstruction of reality

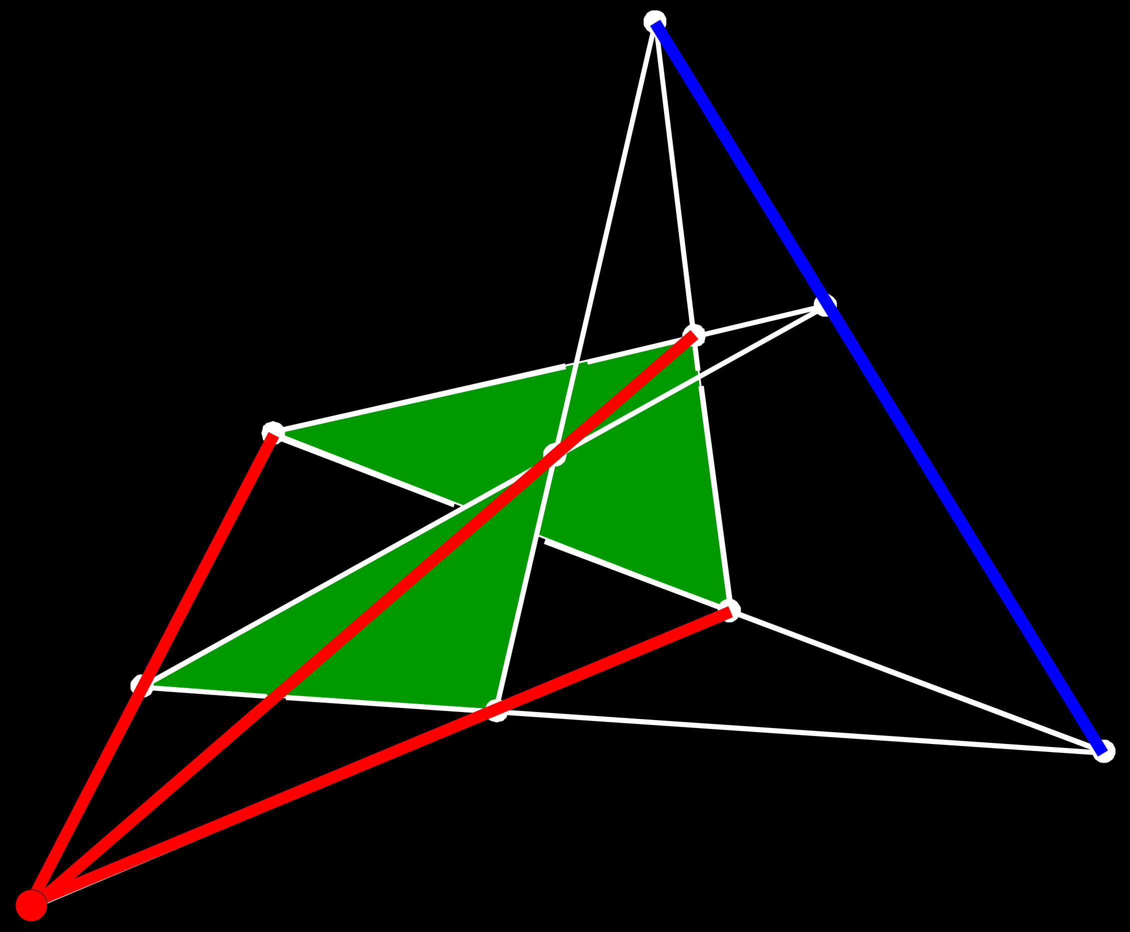
Reconstruct a spatial image of an object from multiple views

- binocular vision
- Medical technology: CT, MRI, OCT, PET, SPECT, US,

Gérard Desargues (1591-1661)



Gérard Desargues (1591-1661)



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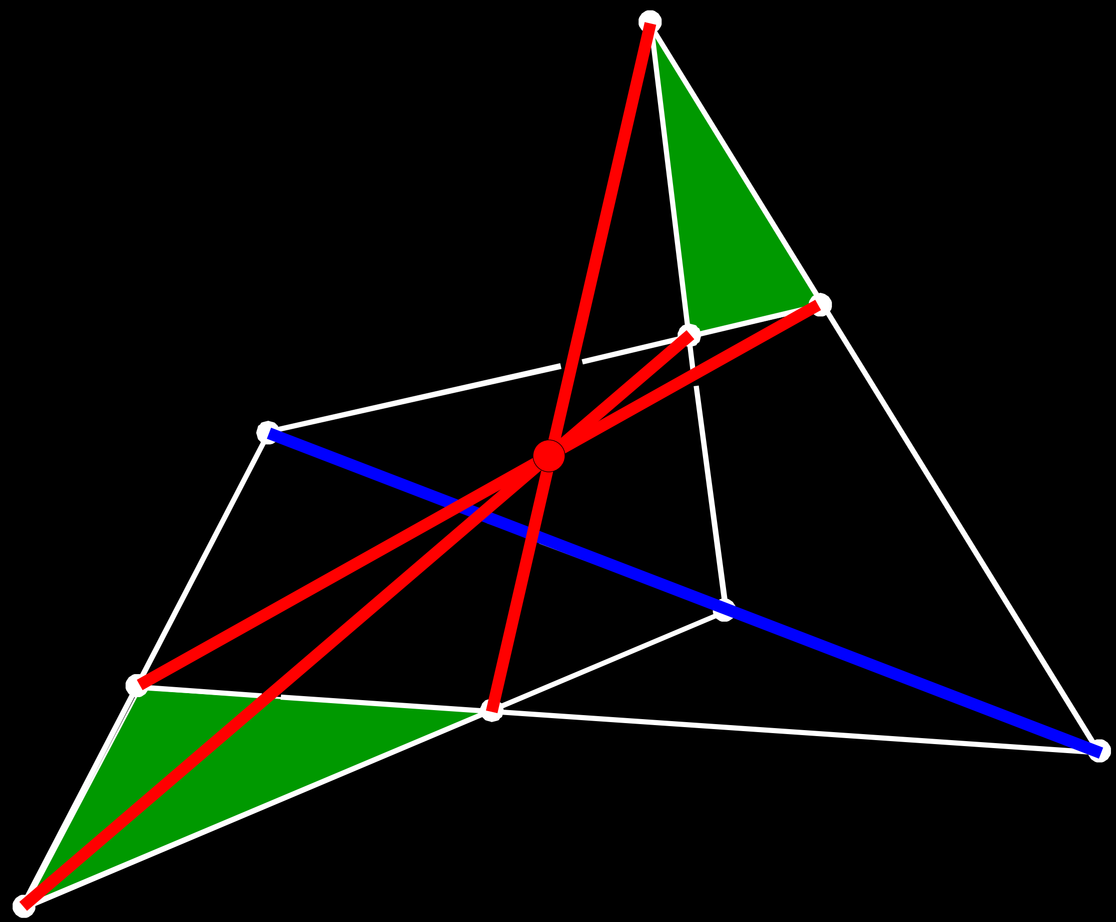
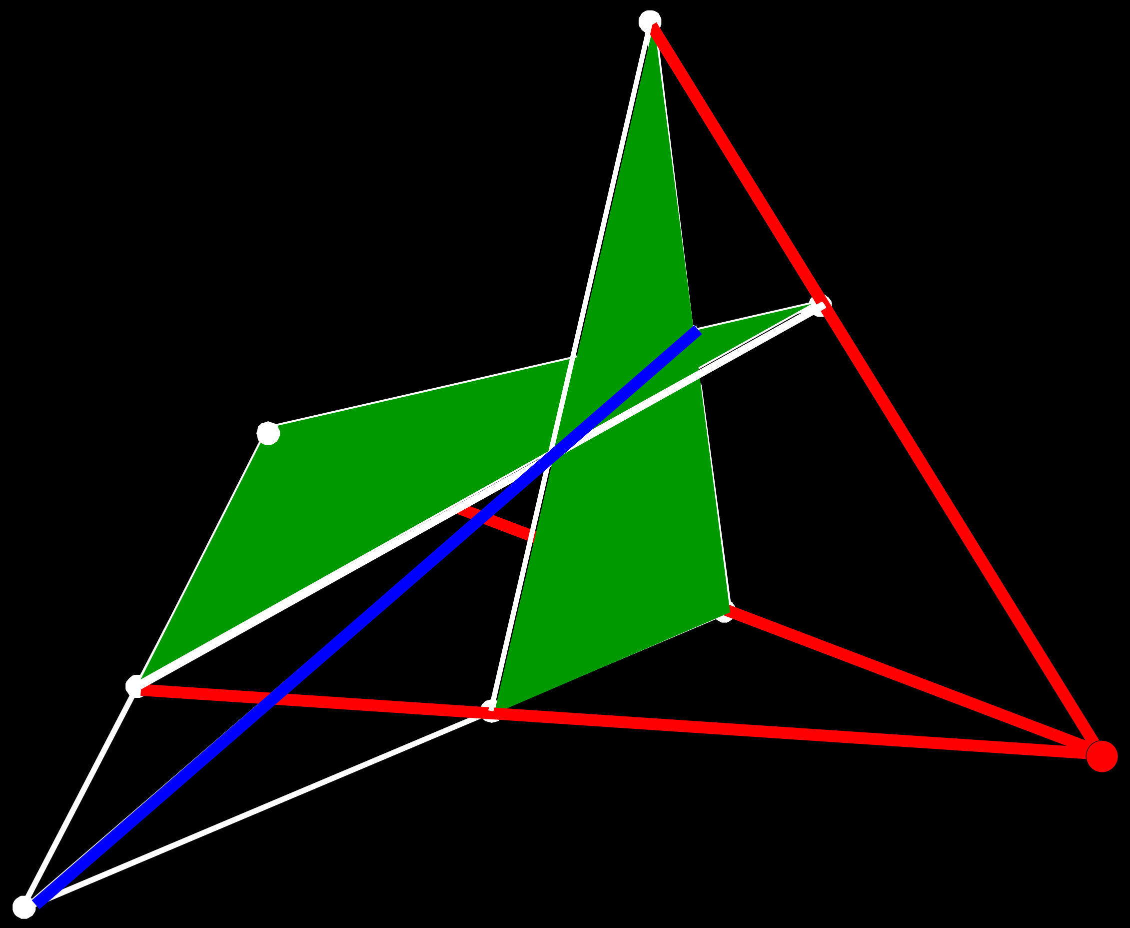


Gérard Desargues (1591-1661)



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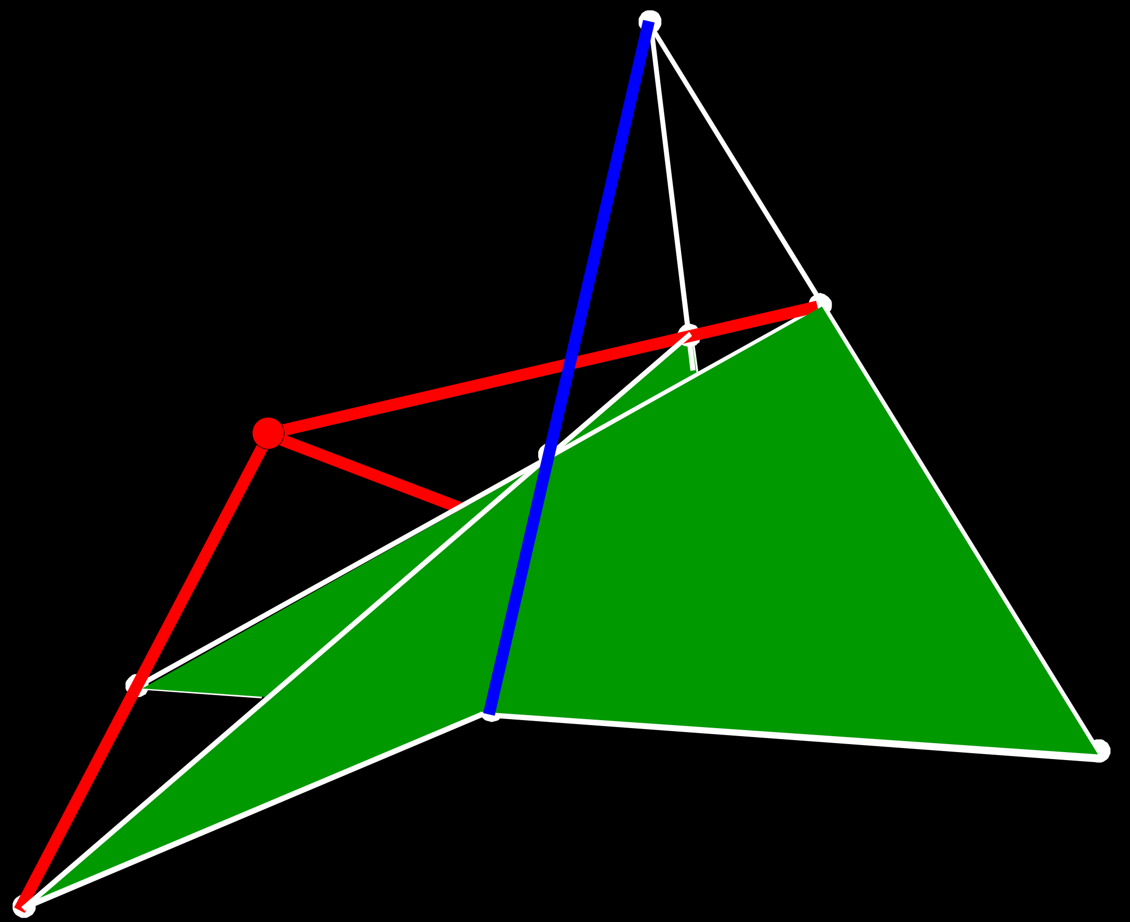

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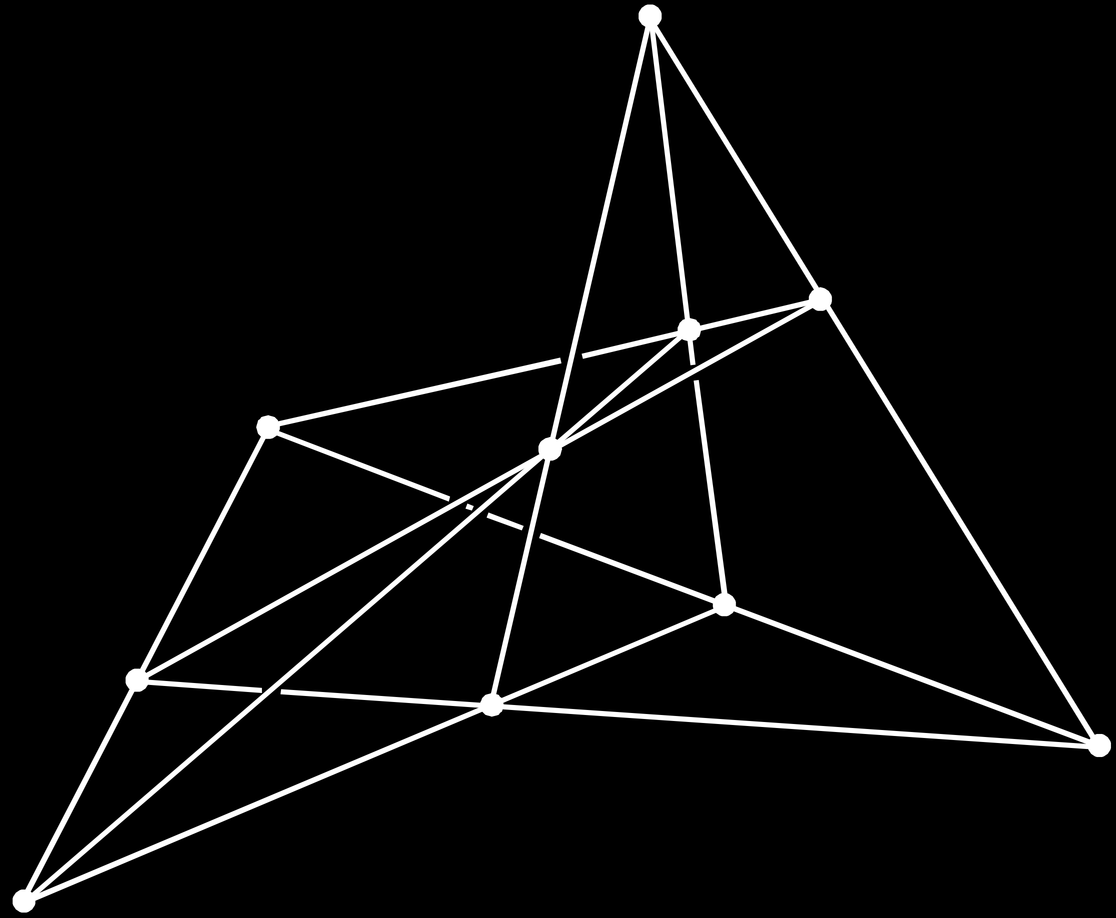


<u>Without</u> point of view: No comprehension of its internal structure

<u>With</u> point of view : Limitation of the diversity of perception

Classical mechanics

System completely characterized by

- **Position** x ***Where?***
- **Momentum** p ***Whither?***
- **Energy structure** H **(Hamiltonian)**

Relation:
equations of motion

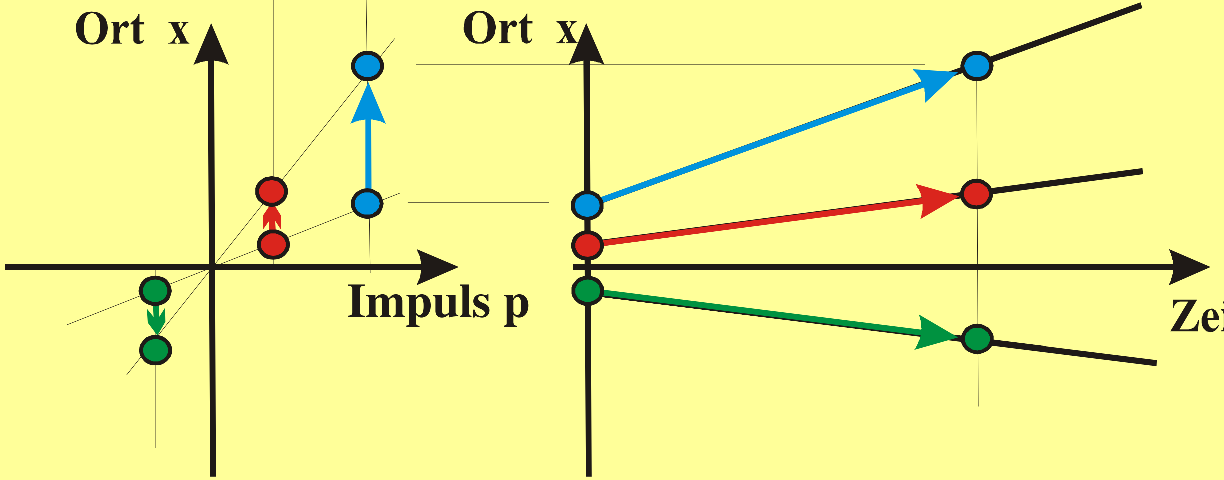
$$\dot{x} = \frac{\partial H}{\partial p} \quad \dot{p} = -\frac{\partial H}{\partial x}$$

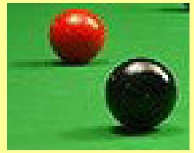
In the case of an oscillation:
Instead of position x and momentum p
Amplitude A ***How large?***
Phase φ ***When?***

Classical mechanics

Free particle

Billards

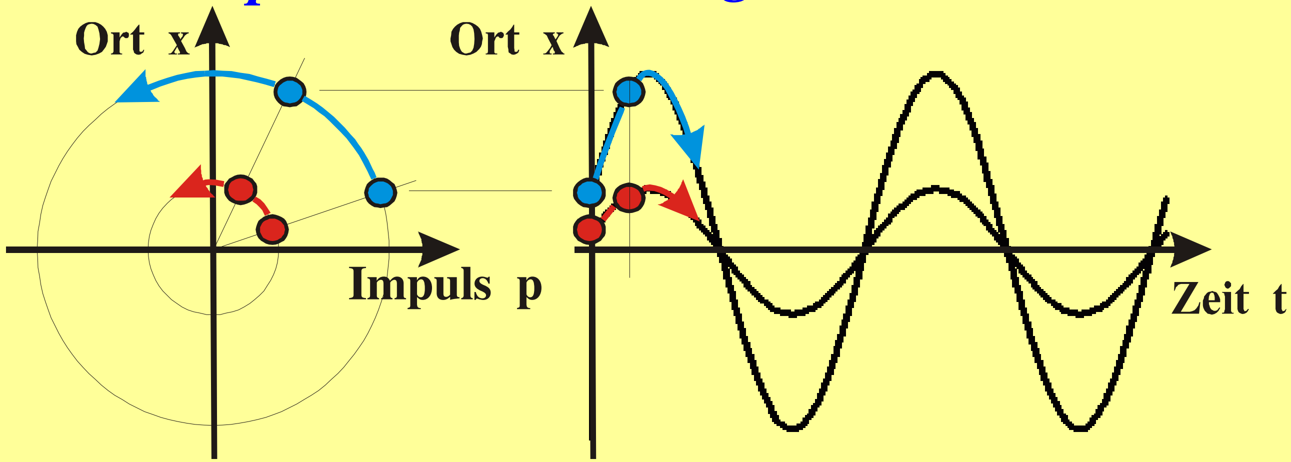


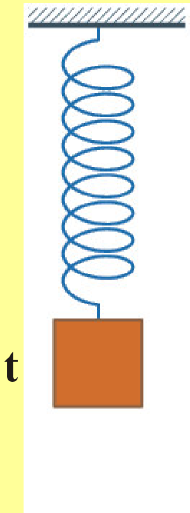


$$H = \frac{p^2}{2m}$$

Phase space

Time diagram





$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2} x^2$$

Harmonic oscillation

Pendulum

Quantum Mechanics

Position and momentum not simultaneously measurable

System completely determined by

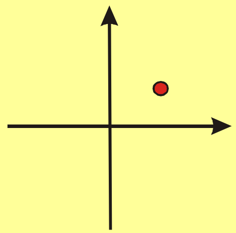
- **Wave function** $\psi(x,t)$ **or** $\psi(p,t)$
- **Energy structure *H* (Hamiltonian)**

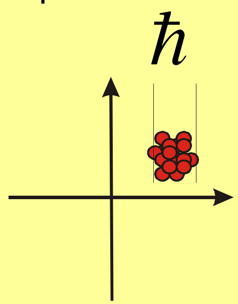
Relation: Equations of motion $$i\hbar \frac{d\psi}{dt} = H\psi$$

Meaning: Position distribution $P(x) = |\psi(x)|^2$

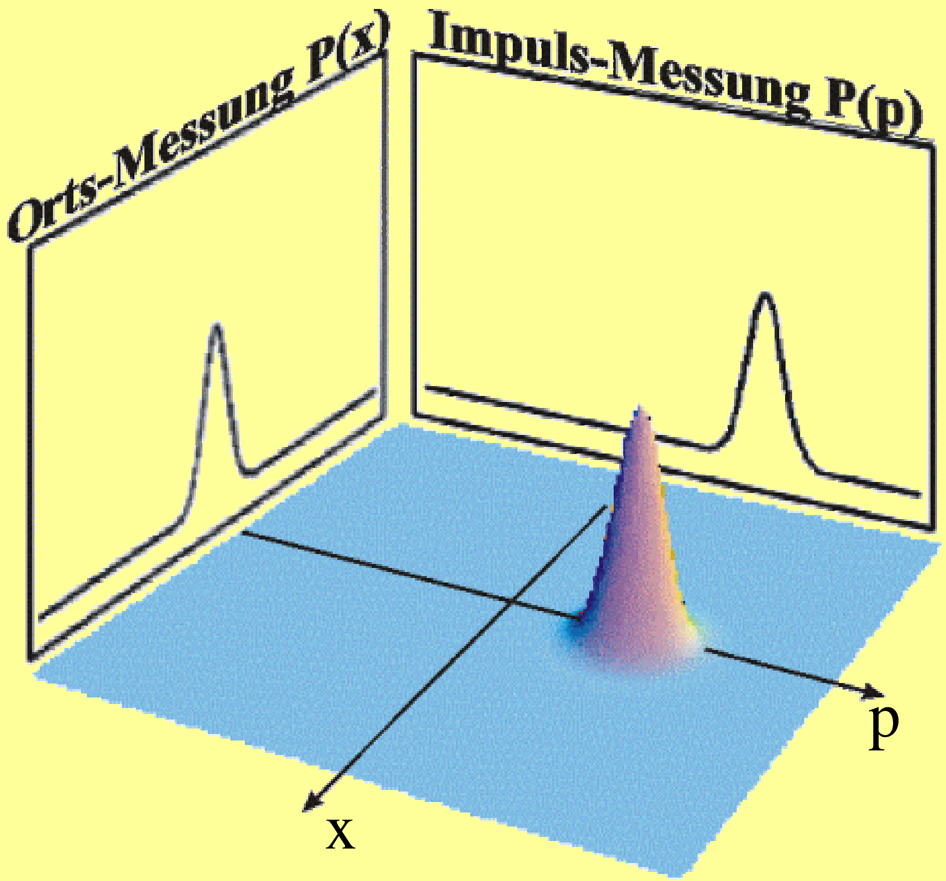
Momentum distribution $P(p) = |\psi(p)|^2$

Consequence in phase space:
Not a point *(x,p)*
But a „stain" *W(x,p)*, a distribution





Measurement of a state by projection



Wignerfunction

Quasiprobability distribution

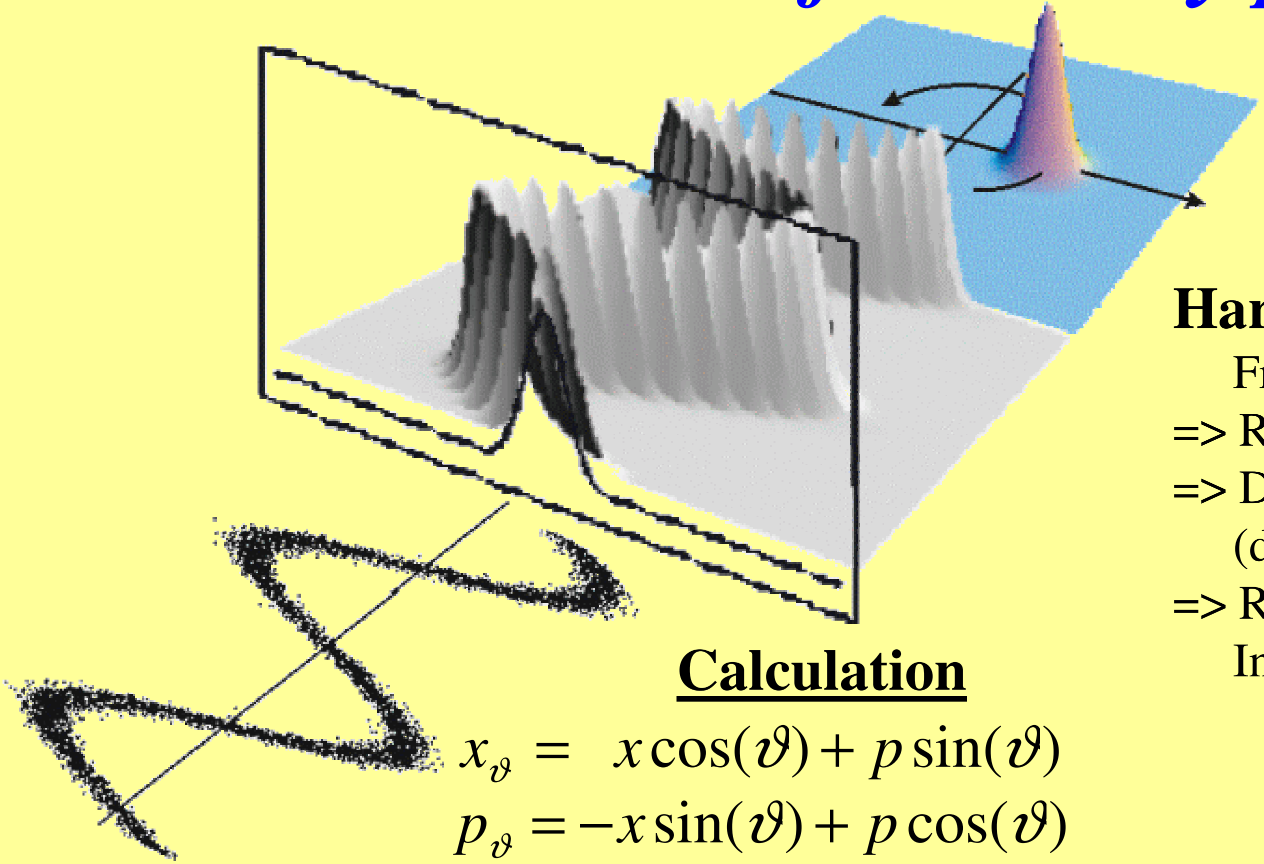
$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty}\psi^*(x+y)\psi(x-y)e^{2ipy/\hbar}dy$$

Marginals

$$\int_{-\infty}^{\infty} W(x,p)\,dp = \left|\psi(x)\right|^2 = P(x)$$

$$\int_{-\infty}^{\infty} W(x,p)\,dx = \left|\psi(p)\right|^2 = P(p)$$

Measurement of a state by projection



Example

Harmonic Oscillator

Free time development
=> Rotation of the state
=> Different „view points“
(density projections)
=> Reconstruction of the state:
Inverse Radon Transformation

Calculation

$$x_\vartheta = \; x\cos(\vartheta) + p\sin(\vartheta)$$
$$p_\vartheta = -x\sin(\vartheta) + p\cos(\vartheta)$$

$$\int_{-\infty}^{\infty} W(x_\vartheta, p_\vartheta)\, dp_\vartheta = P(x_\vartheta)$$

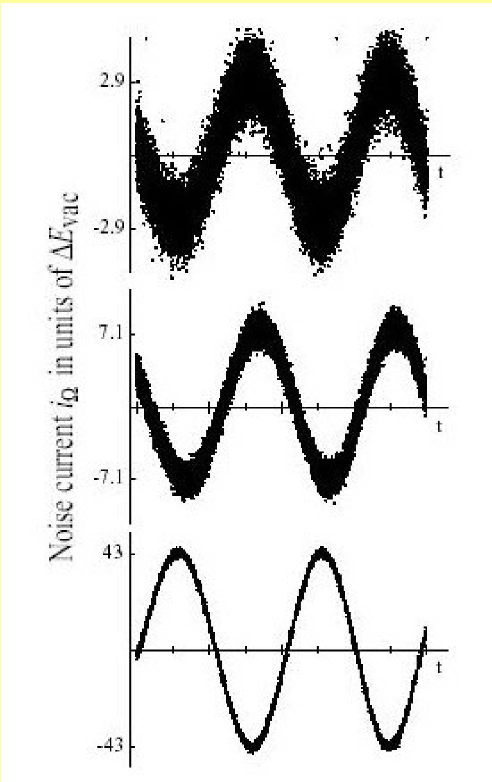
Johann Radon 1917
K. Vogel, H. Risken, PRA **40**, 2847 (1989)
M. Raymer et al., PRL **72**, 2183 (1994)

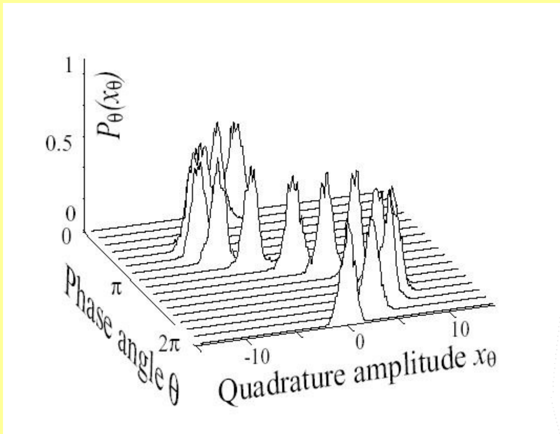
$$W(x,p) = \frac{1}{4\pi^2}\int_0^\pi d\vartheta \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dr\, |r|\, P_\vartheta(x') \exp[ir(x' - x_\vartheta)]$$

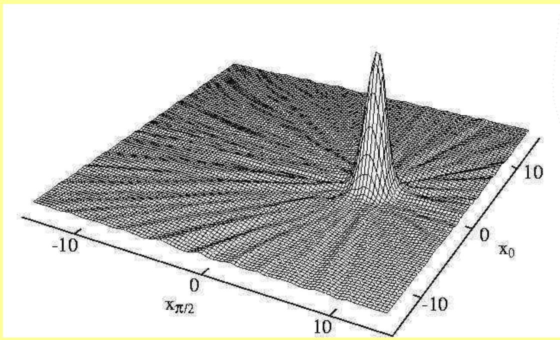
Experiment:Measurement of light waves

Classical oscillation: Sine curve
Quantum mechanics: Unvoidable phase and amplitude noise
Measured: The electric field of a very weak light wave

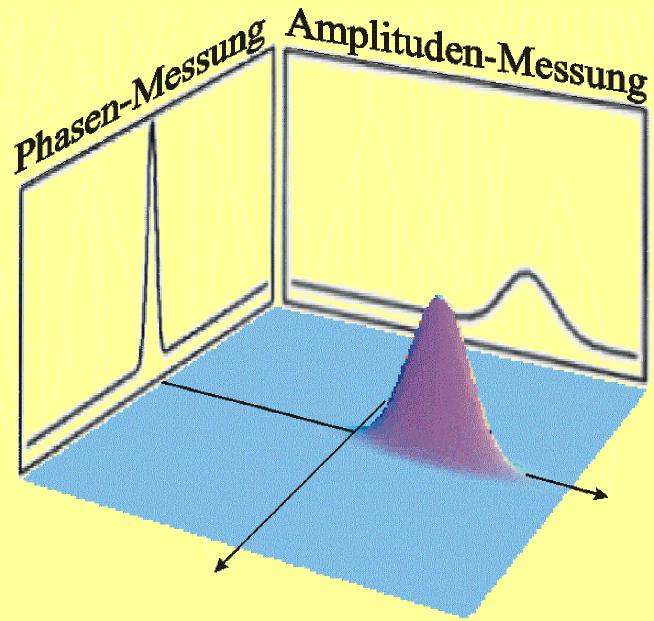
Ideal Laser: Coherent State





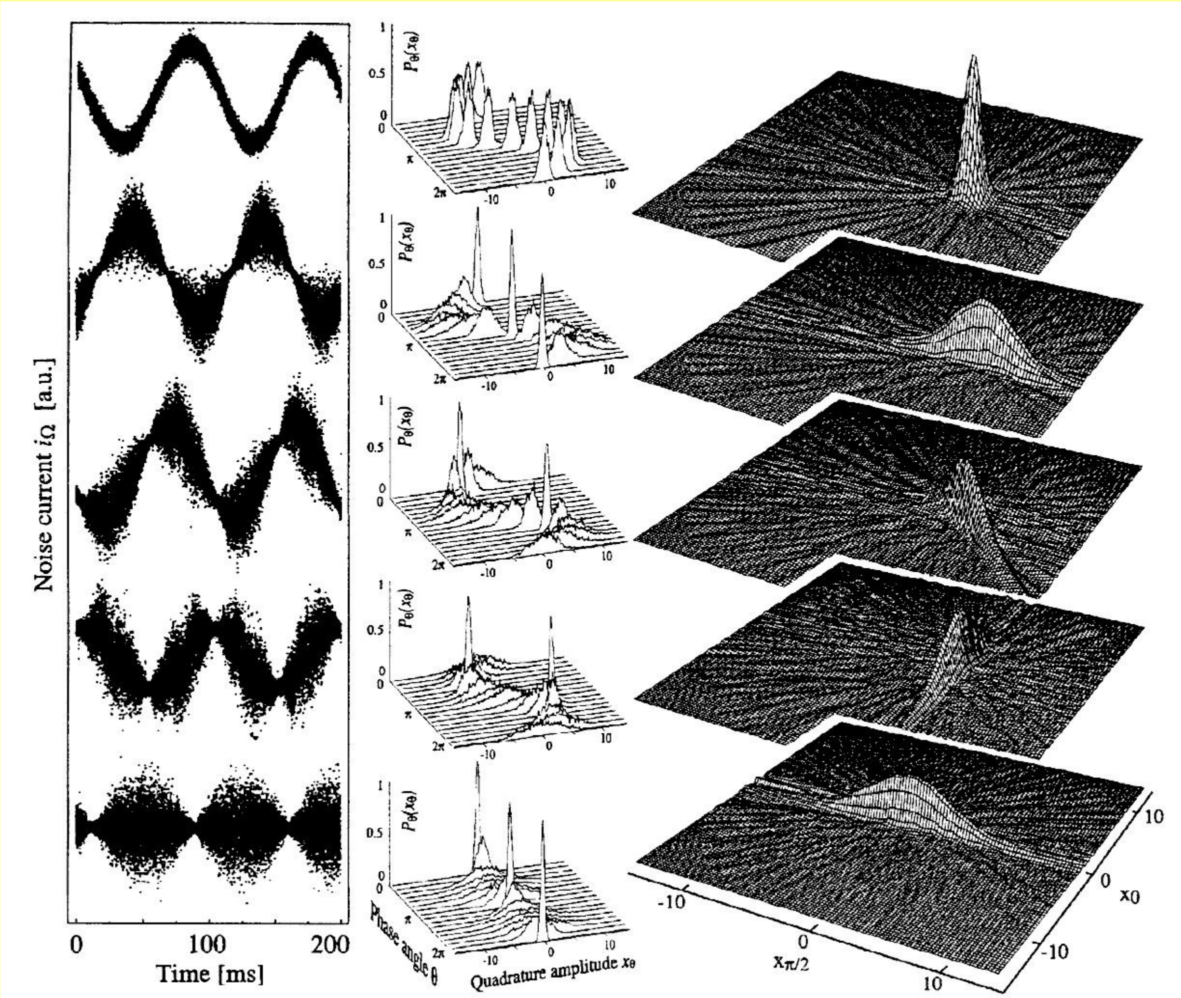


But: Uncertainties need not be evenly distributed

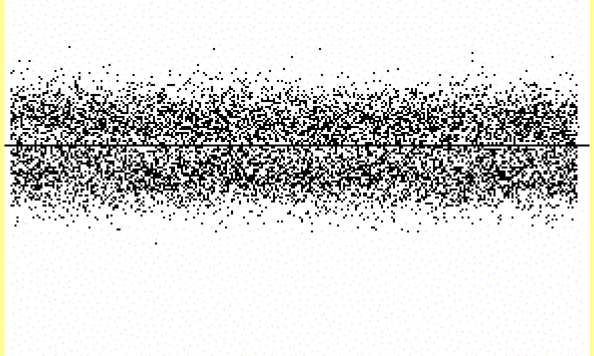


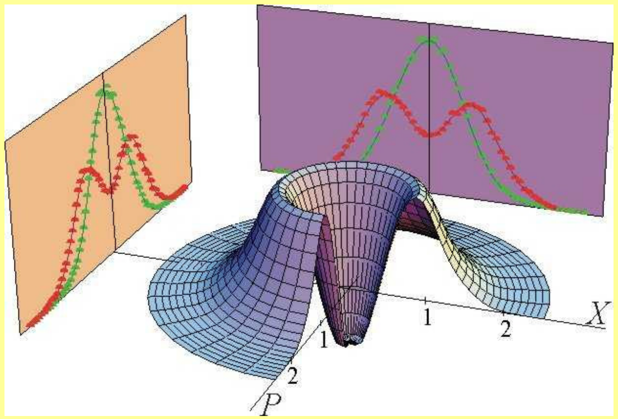
Experiment: Measurement of light waves

Preparation of light waves by nonlinear crystals



Ein-Photon-Zustand

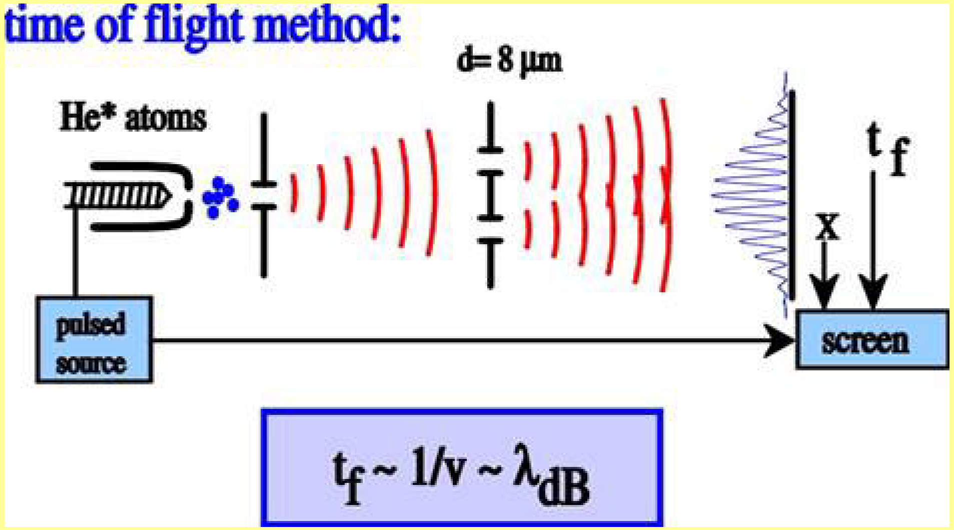


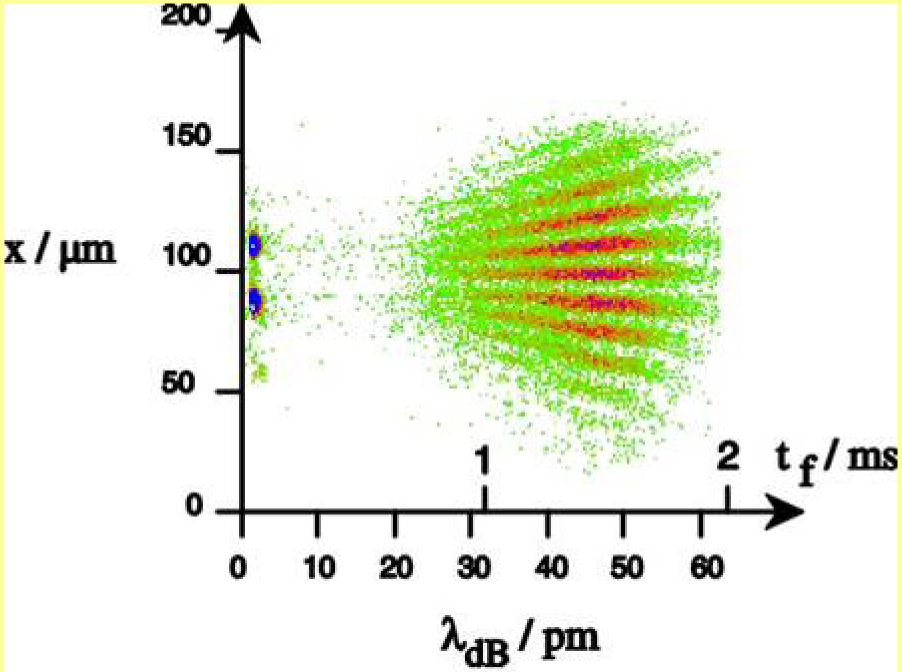


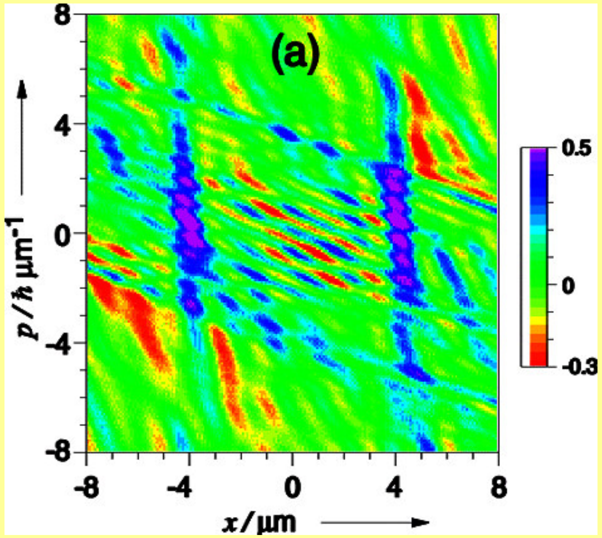
A.I. Lvovsky et al. Phys. Rev. Lett. 87, (2002);

G. Breitenbach, S. Schiller, and J. Mlynek, Nature, 387, 471 (1997);
Lvovsky, Raymer, Rev. Mod. Phys., 81, 2009

Experiment: Diffraction of helium atoms



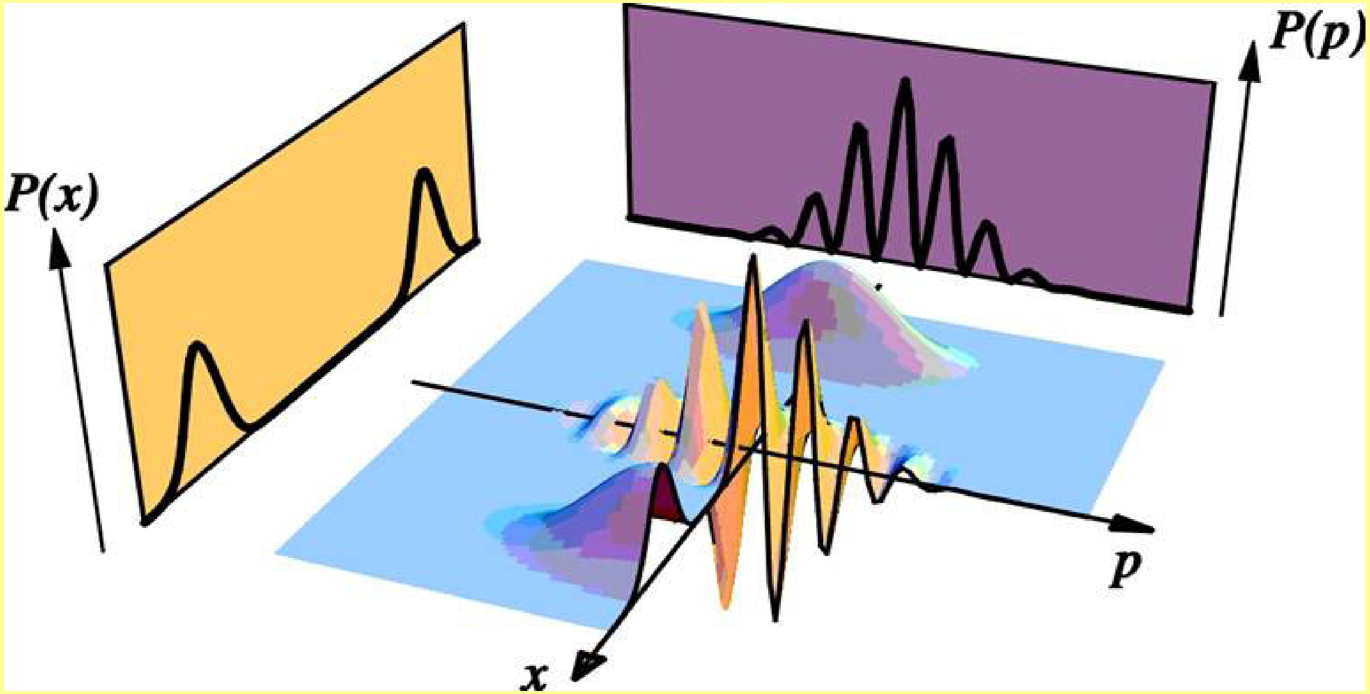




Ch. Kurtsiefer, T. Pfau, J. Mlynek, Nature **386**, 150 (1997)

Experiment: Diffraction of helium atoms

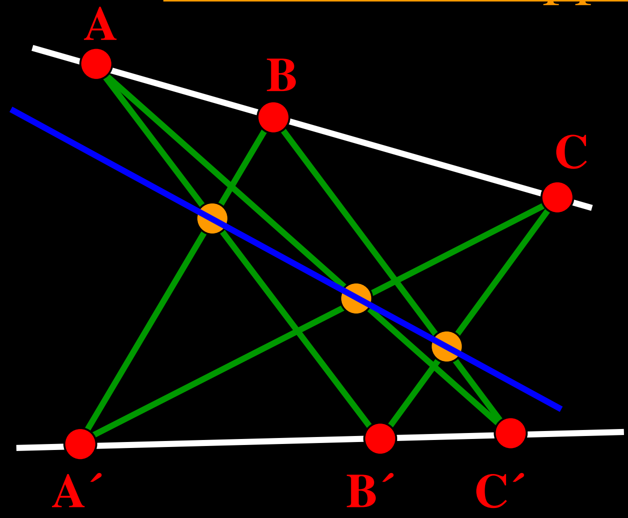
Wignerfunction of an atom directly at the double slit
Depending on view information about
possible position or inherent interference capability

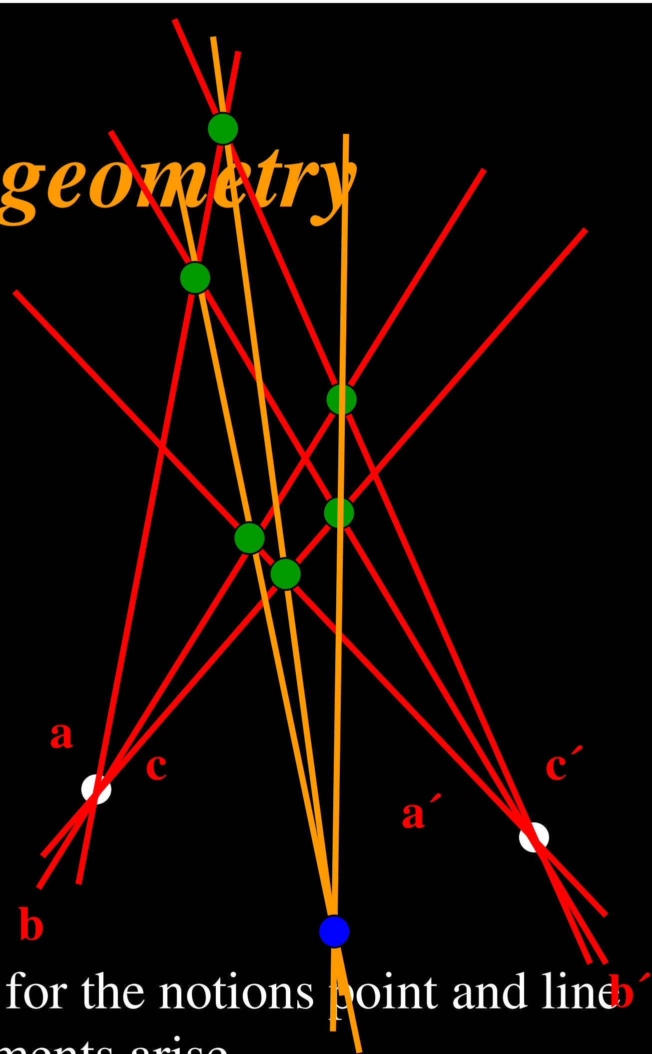


Distribution function with negative regions

Duality, projective geometry

Theorem of Pappos





plane projective geometry: Symmetry of axioms for the notions point and line

⇒ by switching the notions new geometric statements arise

⇒ notional change of viewpoint (D. Hilbert)

Mathematical content: The same

Realization in the imagination: Completely different

What is reality?

Duality, quantum mechanics

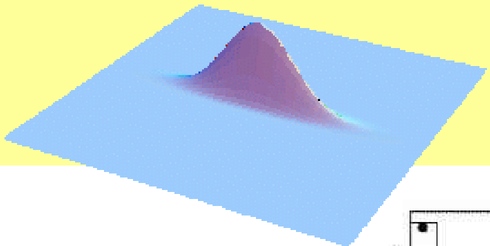
Switch basis $|x> \;\rightarrow\; |n>$ position to energy

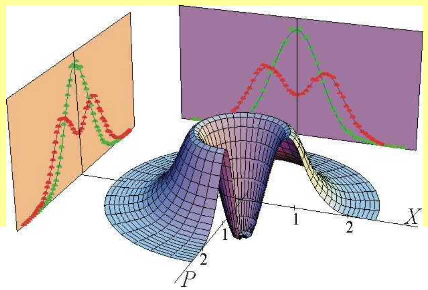
Relation: $< x|n > = \dfrac{1}{\sqrt[4]{\pi}\sqrt{2^n n!}} H_n(x)\exp(-x^2/2)$ (H_n Hermite)

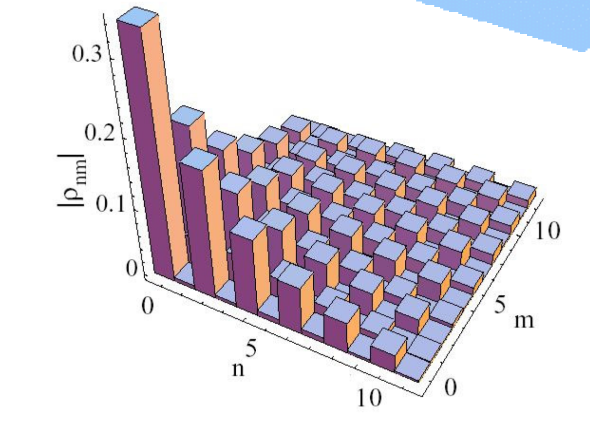
Classical analogon: Decomposition of a movement into oscillations

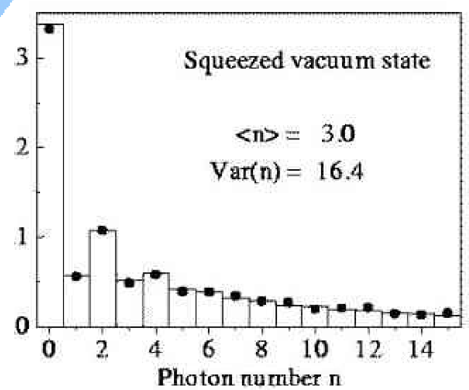
Switch notions: Amplitude, phase → photon number

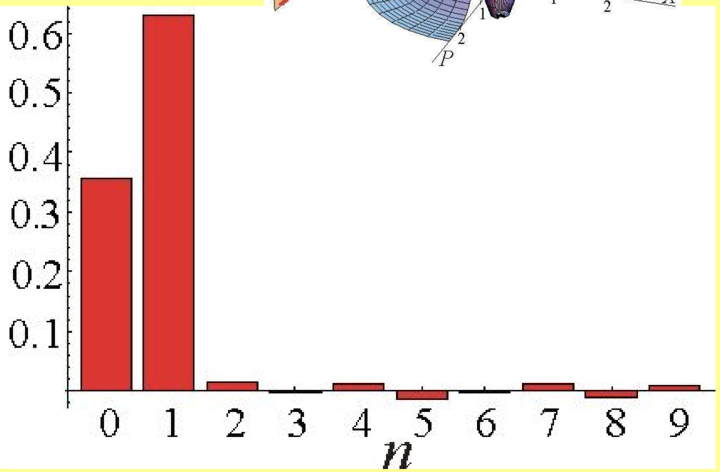
Wignerfunction and density operator in Fock space











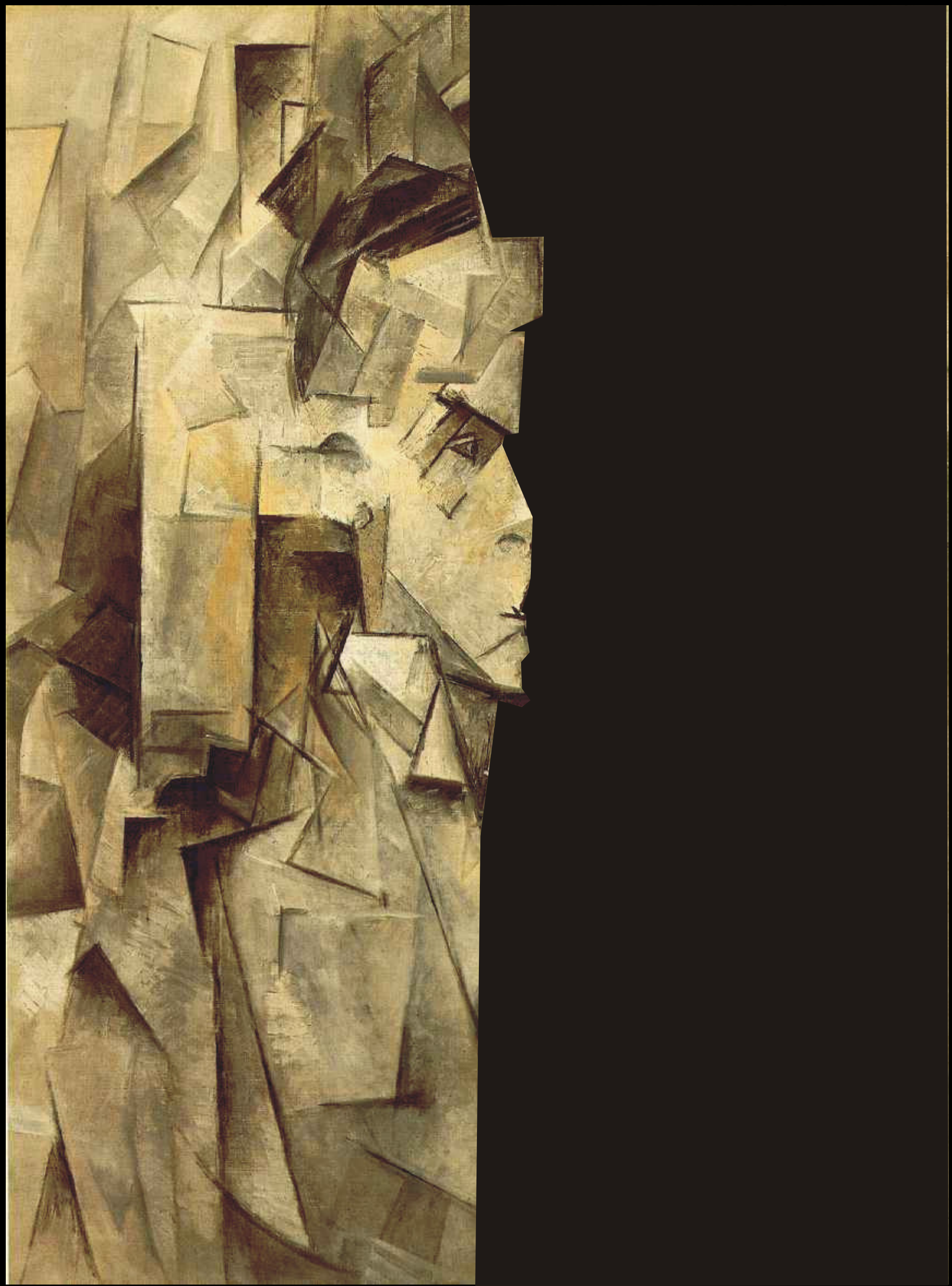
Summary

Classical perspektive: Attempt to depict an unambiguous world
independent of the observer

Quantum mechanics: Observation = unique projection
Result of a measurement / perception of the world
depends on measurement device and orientation

Reality, a completely personal matter?
Yes: The existence of different equivalent reality concepts is experiencable in geometry by change of perspektive
No: Not all perspektives are realizable (ex.: triangle / quadrangle)

Cubism: Attempt to overcome the limitations of perspective
Depict simultaneously a multitude of viewpoints in one painting
Challenge the viewer to an independent reconstruction



1906

Pablo Picasso (1881-1973)
Portrait Wilhelm Uhde
1910

1915

Pablo Picasso (1881-1973)
Portrait Ambroise Vollard
1909 / 10

Foto David-Henry Kahnweiler 1912

Juan Gris
Sketch David-Henry Kahnweiler

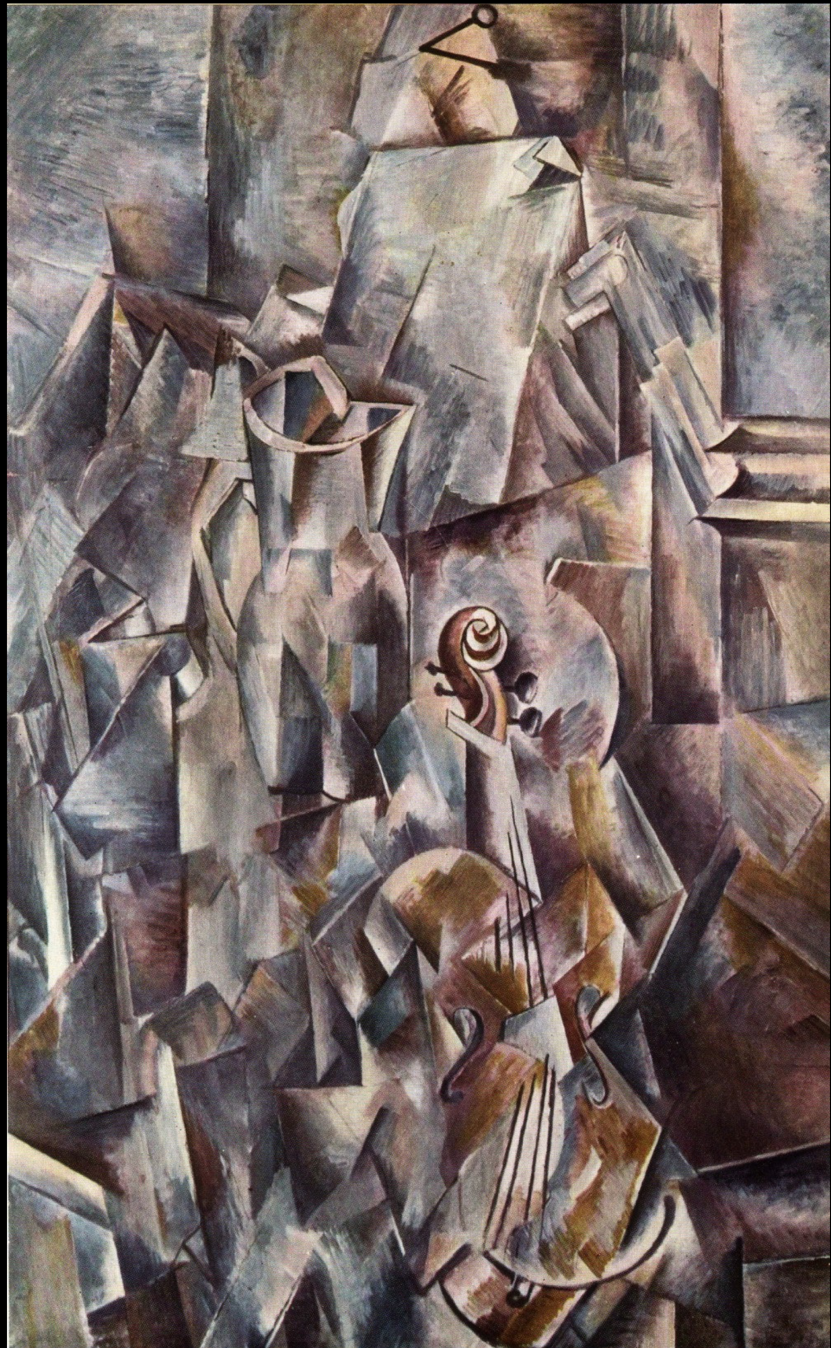
Pablo Picasso (1881-1973)
Portrait David-Henry Kahnweiler
1910

Juan Gris (1887–1927)
Portrait Picasso, 1912

Our heads are round so our thoughts can change direction.

Francis Picabia (1879-1953)

End



George Braque (1882–1963)
Violine und Krug   1912

Marcel Duchamp (1887–1968)
Akt, eine Treppe herabsteigend
1912

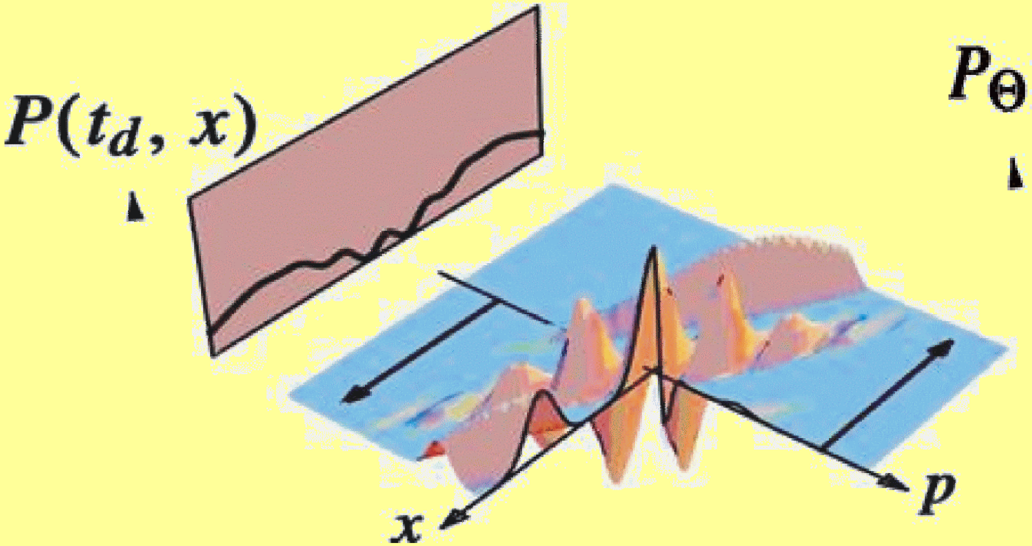
Kasimir Malewitsch (1878-1935)
Der Scherenschleifer
1906

Pablo Picasso (1881-1973)
Les Desmoiselles d'Avignon
1906

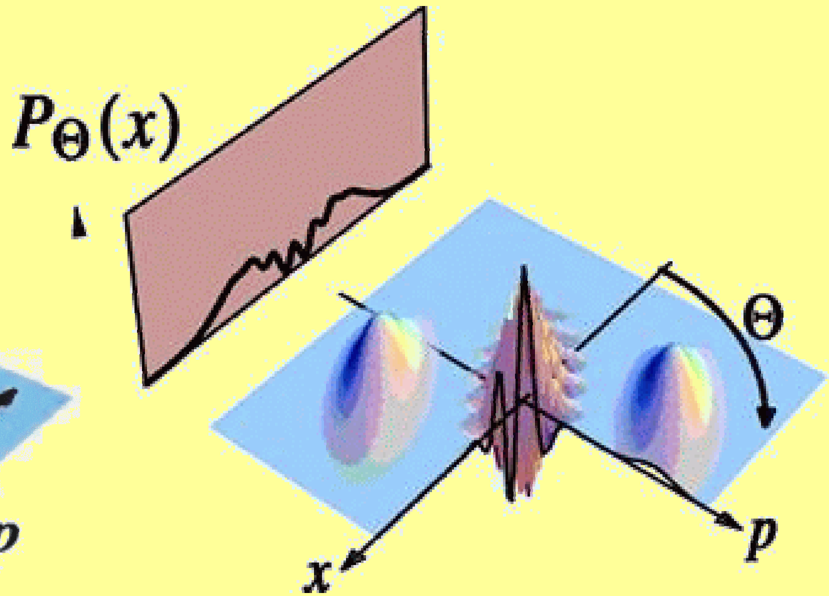
Measurement of a state by projection

Different perspectives

Free particle
spreading / shearing

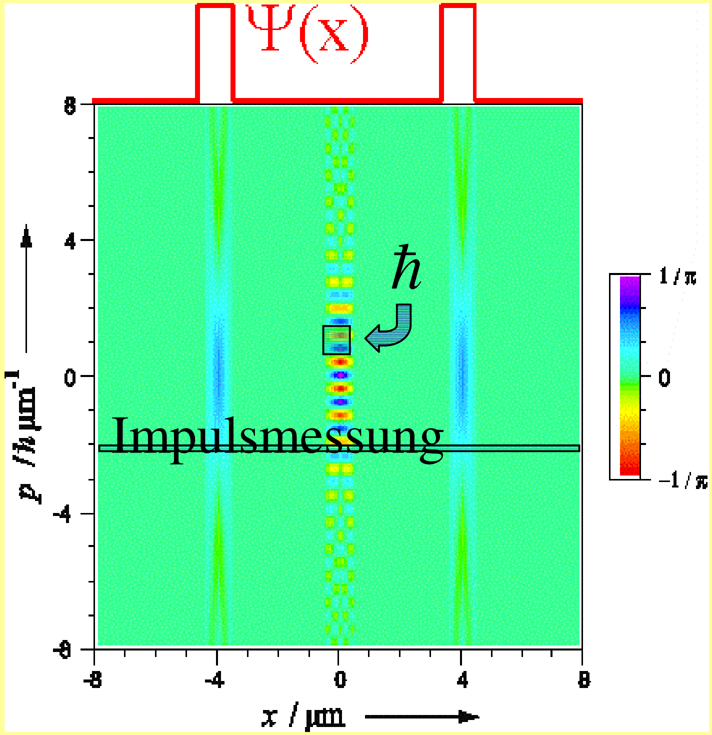


Harmonic oscillator
rotation

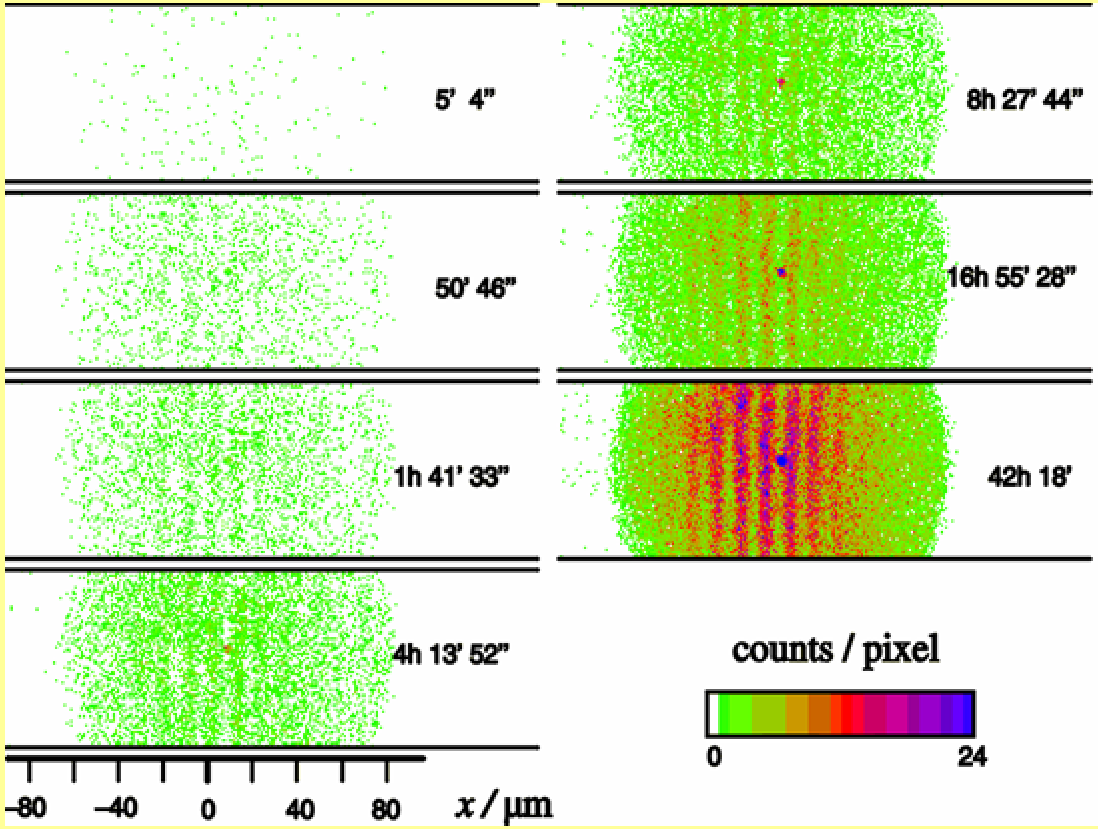


Experiment: Diffraction of helium atoms

Details theory

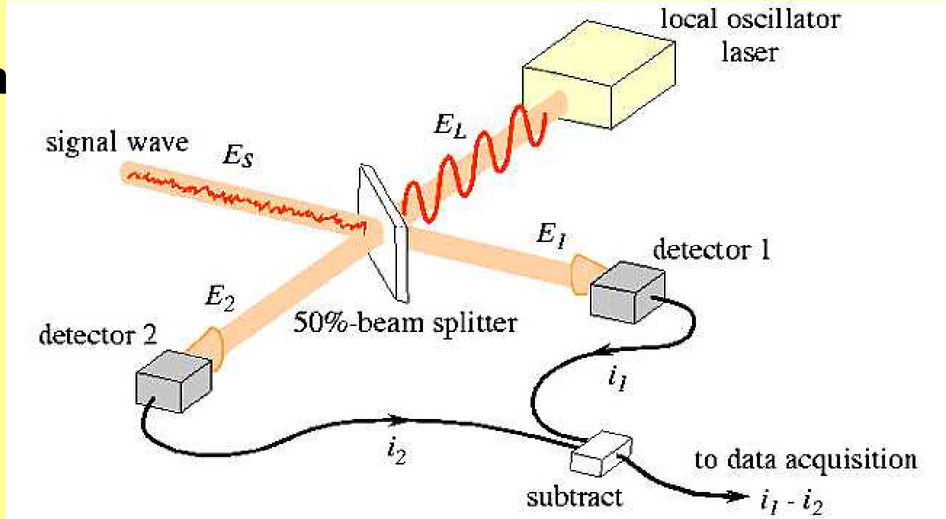


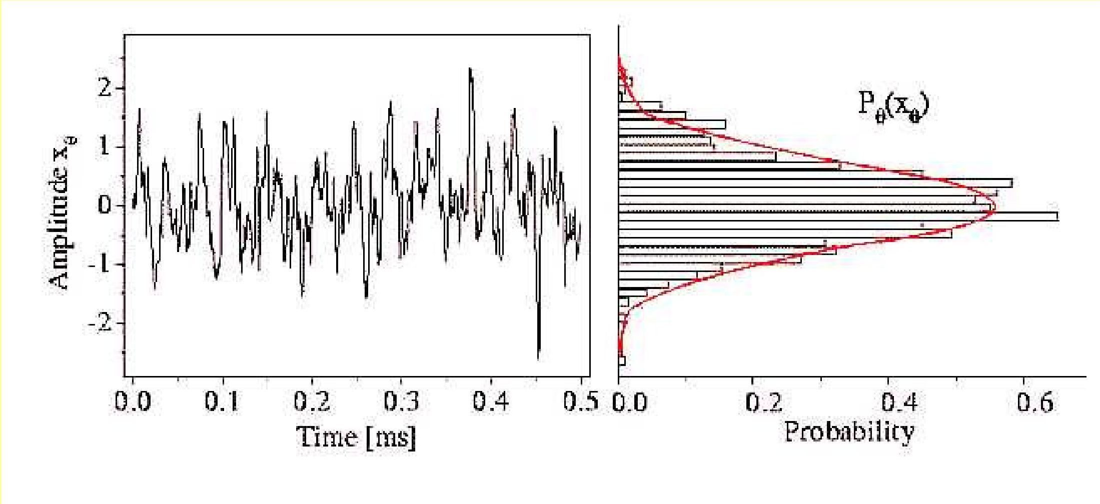
Emergence of the diffraction pattern



Light wave experiment

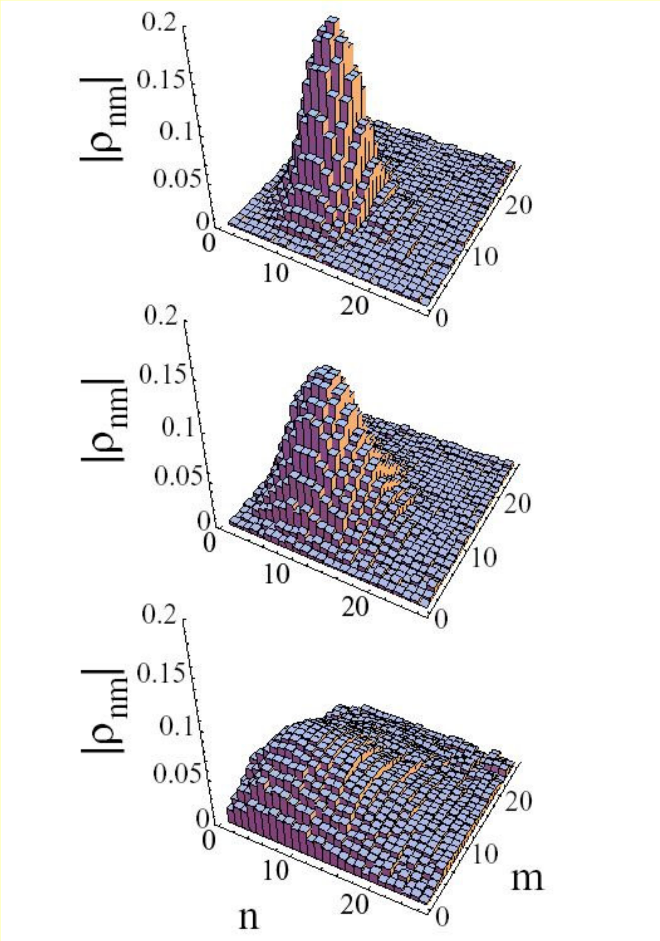
Homodyne system

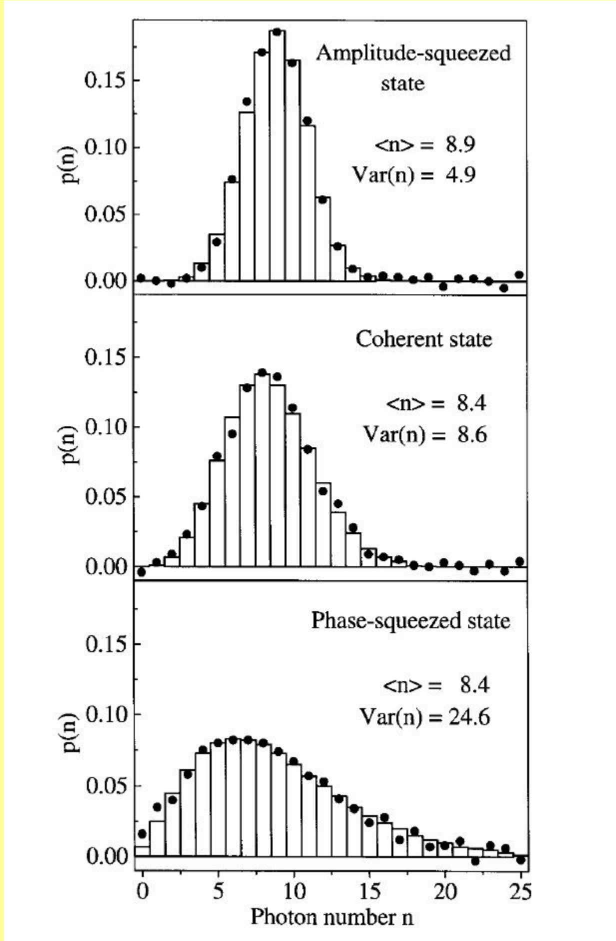




Light wave experiment

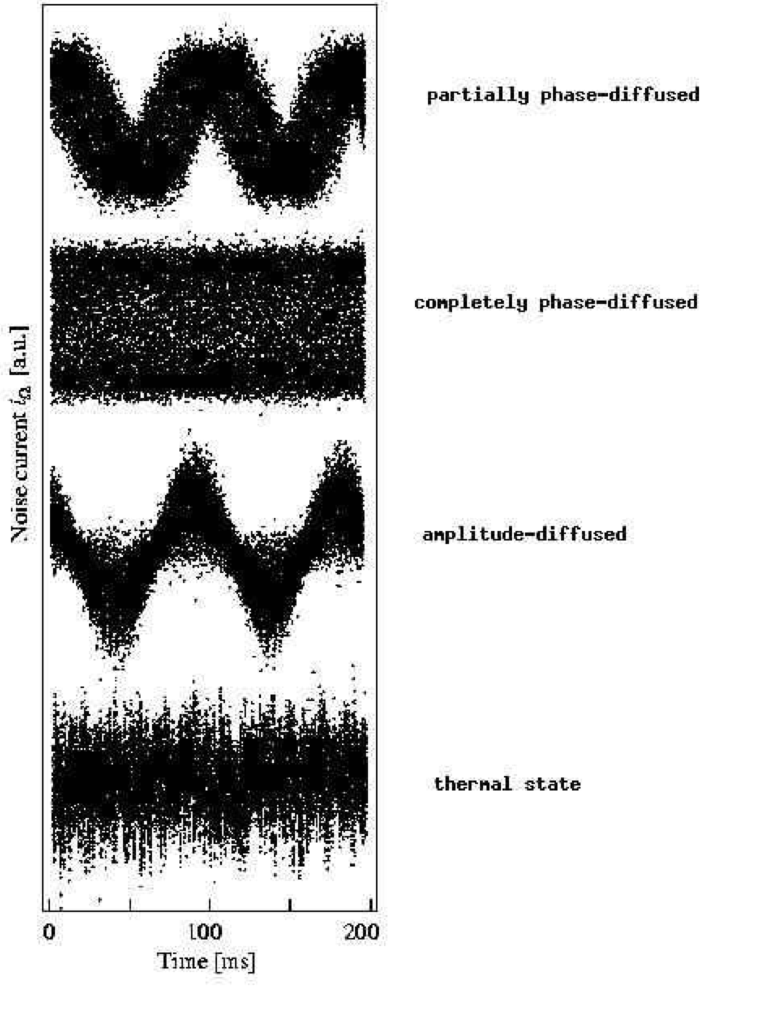
density operators and photon statistics I

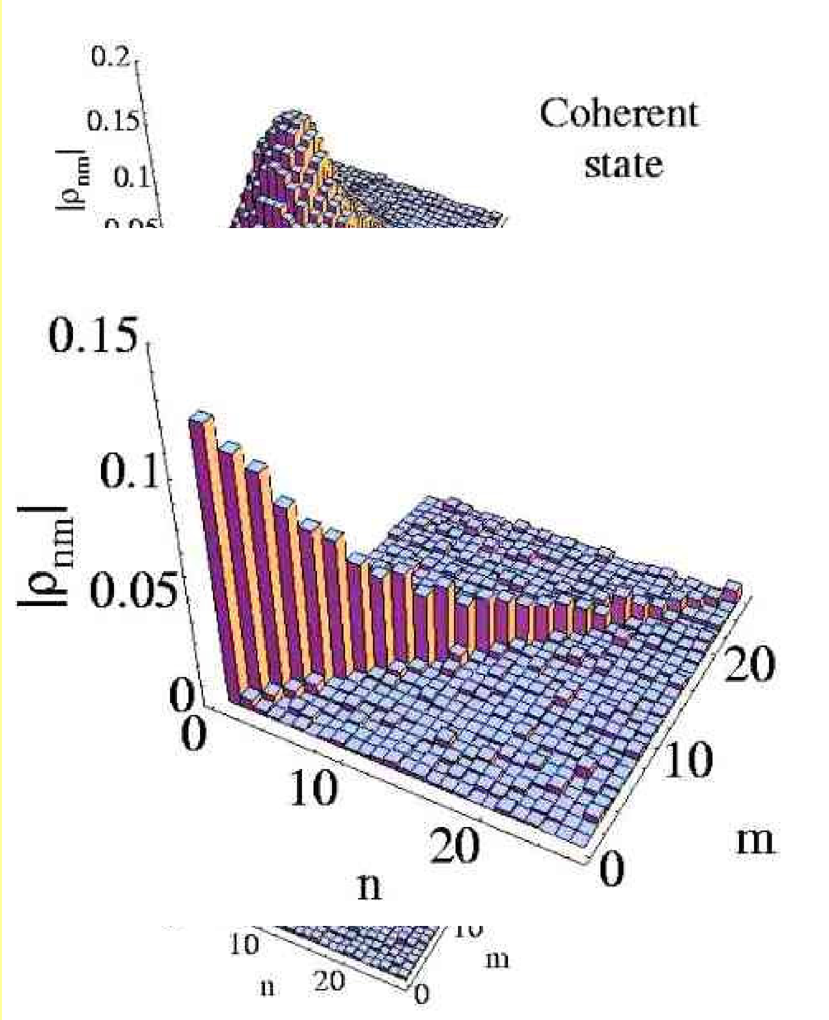




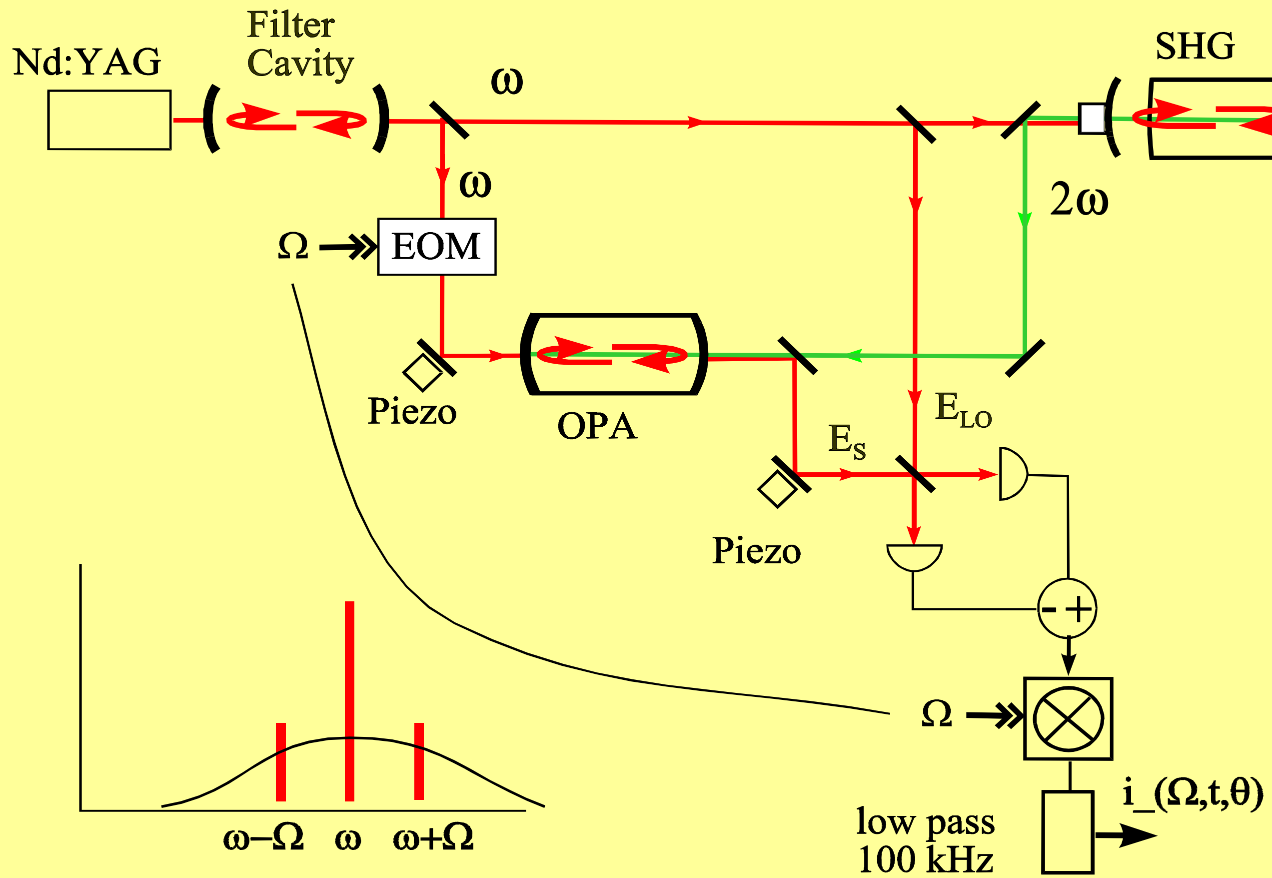
Light wave experiment

density operators and photon statistics II

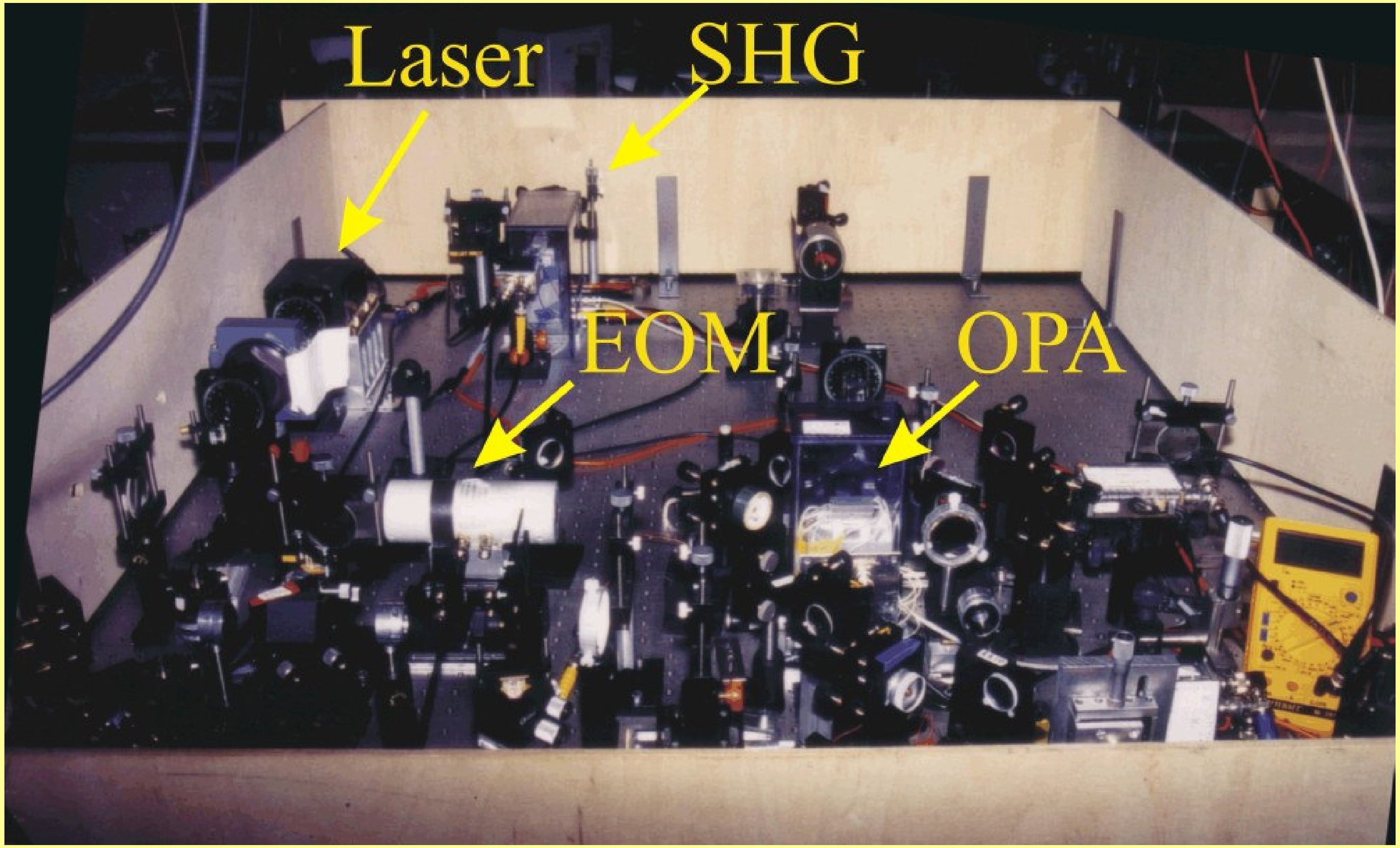




Light wave experiment: Setup



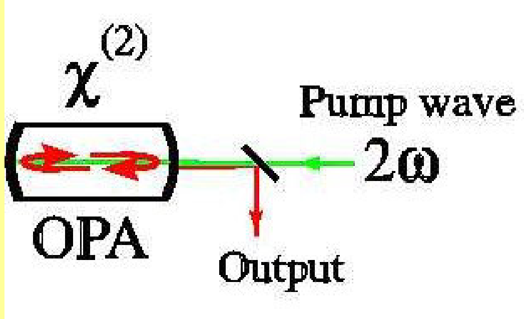
Light wave experiment: Laboratory

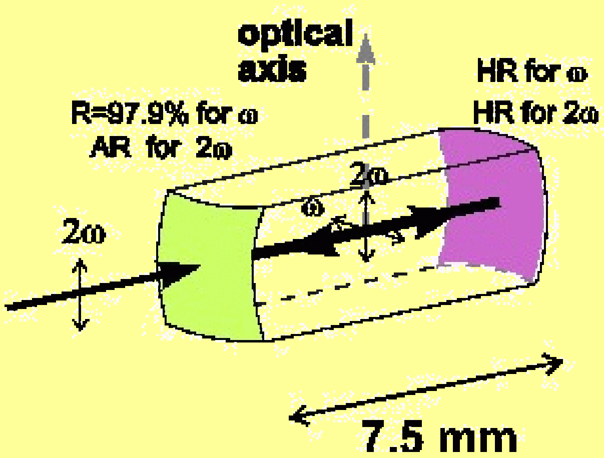


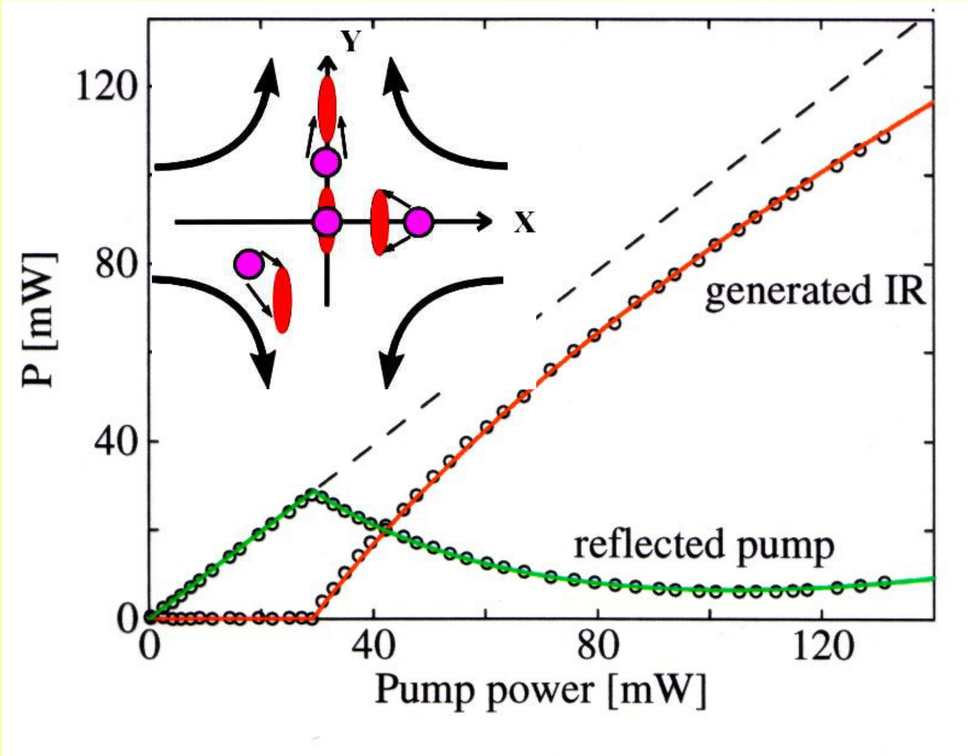
Light wave experiment

Preparation of the light wave by nonlinear media

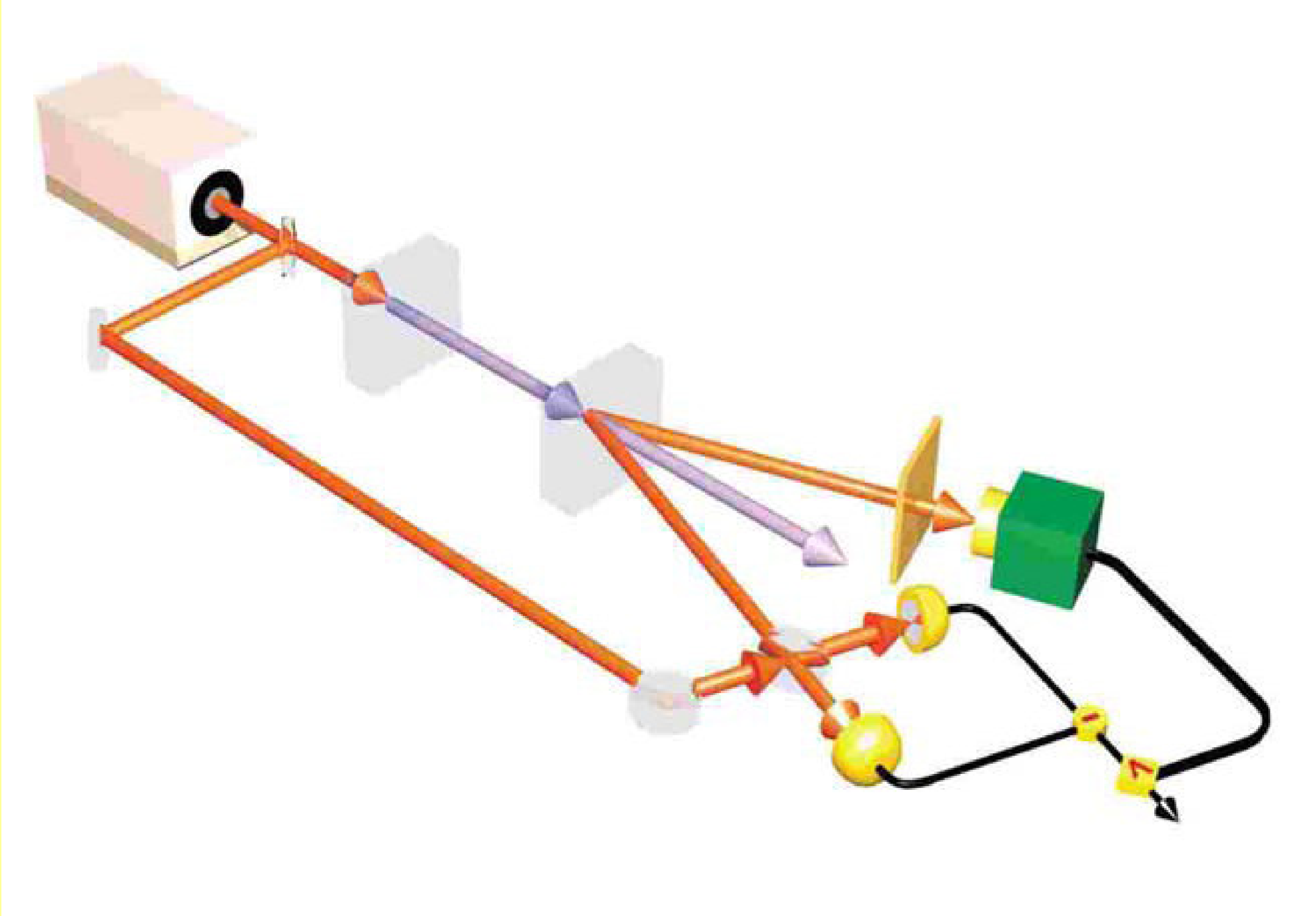
Highly efficient resonante frequency conversion (> 80 %)
532 nm ⇨ signal/idler at 1064 nm cw







Light wave experiment:
Scheme 1-Photon-Measurement



Light wave experiment: The Reality